MODELING AND MULTI-OBJECTIVE OPTIMIZATION FOR ENERGY-BASED TARGET LOCALIZATION USING WIRELESS SENSOR NETWORKS

by

ZHENXING LUO

THOMAS C. JANNETT, CHAIR
B. EARL WELLS
GREGG L. VAUGHN
GREGORY A. FRANKLIN
IAN KNOWLES

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ZHENXING LUO

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ABSTRACT

This dissertation focuses on energy-based target localization in wireless sensor networks (WSNs). A WSN consists of a fusion center and a large number of sensors. The fusion center localizes a target using a maximum likelihood estimation (MLE) method based on decisions received from sensors whose positions are known. However, several factors may degrade localization performance, including imperfect communication channels, sensor faults, sensor position uncertainty, and sensor sleep. Localization performance may be improved if these factors are accounted for in the MLE framework.

In this dissertation, models were developed that describe how localization performance is affected by imperfect communication channels, multi-hop communication channels, sensor position uncertainty, sensor faults, and sensor sleep. These models were incorporated into the MLE framework either simultaneously or separately. The Cramer-Rao lower bound on the estimation error (CRLB) was also derived. Simulation results showed that the new MLE methods gave better localization performance in terms of root mean square (RMS) estimation errors than current methods which ignore these factors that degrade localization performance. Moreover, the RMS estimation errors were close to the CRLB.
Based on the sensor sleep model, an energy saving strategy was developed in which the sensor sleep probability is used to adjust energy consumption of the WSN. However, although the sensor sleep strategy saves energy, sleeping sensors cannot send decisions to the fusion center and missing decisions can degrade localization performance. Therefore, the system design must consider tradeoffs between energy consumption and localization performance. Models of energy consumption and localization performance were developed for one-dimensional and two-dimensional sensor arrays. These models were used in a multi-objective optimization method that balances energy consumption and localization performance. The Pareto-fronts generated by the multi-objective optimization method allow the system designer to make tradeoffs between energy consumption and localization performance.
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I. INTRODUCTION

Recently, the ability of sensors to monitor environmental phenomena has been enhanced by forming a network of a large number of sensors. Sensors typically include an electronic circuit for interfacing with measurement units, a microcontroller, a transceiver with an antenna or wired connection, and an energy source [1]. The sensor network may use a fusion center to gather and process information from sensors. In ad hoc networks, there is no fusion center; each sensor has equal status, and each sensor forwards data to other sensors [2]. If communication channels are wireless, the sensor network is called a wireless sensor network (WSN). WSNs have been used in many areas including environmental monitoring [3]-[6], machine health monitoring and structure monitoring [7][8], health care [9]-[14], fire rescue, humanitarian search and rescue system [15][16], and traffic and transportation management [17]-[19].

WSNs can carry out many tasks that are important in surveillance applications, including target detection, target localization, and target tracking. In carrying out these tasks, the fusion center first gathers information from sensors and then processes information to provide a higher level of intelligence. For example, in target detection, based on decisions from sensors, the fusion center makes decisions about the presence of a target. In target localization, the fusion center estimates the target position. In target tracking, the fusion center continuously estimates the position of a
moving target. Since the research in this dissertation focuses on target localization, target localization methods will be briefly reviewed in the following paragraphs.

Methods for target localization include time delay of arrival (TDOA), direction of arrival (DOA), and energy-based target localization. DOA methods use direction information to localize a target. Particularly, one category of DOA methods relies on beam-forming to acquire direction information [20-22]. However, the beam-forming method usually assumes that waves come from far field and arrive in parallel, which may not be true in many applications. Another approach is to use directional sensors, which can measure the direction of arriving waves. However, this method puts additional requirements on sensors.

TDOA methods rely on the knowledge of differences in the times at which signals arrive at sensors. After collecting information from sensors, the fusion center can localize the target. Recent advances in TDOA methods can be found in [23-25]. However, TDOA methods require accurate synchronization among sensors, which adds additional cost to sensors. Obtaining accurate timing information is particularly difficult in WSNs that use low-cost and resource-constrained sensors.

Energy-based target localization methods employ signal strength information received from sensors to localize a target. Compared with other methods, such as DOA or TDOA, energy-based target localization methods are popular because they do not require time synchronization information. Therefore, energy-based target localization methods do not put high demands on sensors and are suitable for implementation in WSNs where sensors are low-cost and resource-constrained. The
next section presents a review of the energy-based target localization methods that are relevant for the work in this dissertation.

**Literature Review: Energy-Based Target Localization**

Energy-based target localization methods in which sensors send information to the fusion center using analog signals were described in [26-28]. However, sending analog signals to the fusion center wastes sensor resources, such as communication bandwidth and energy [29]. A maximum likelihood estimation (MLE) approach for energy-based target localization using quantized data was presented in [29]. By transmitting quantized signals instead of analog signals, WSNs can save communication bandwidth and energy without significantly sacrificing localization performance.

The MLE approach for energy-based target localization using quantized data in [29] provides the basis for the research in this dissertation. To illustrate this approach, an exemplary field of 441 sensors uniformly deployed with sensors placed 9 m apart on a grid of the $x$ and $y$ directions is presented (Figure 1). This approach may also be applied to other sensor deployments, including fields in which sensors are placed at random locations. In Figure 1, sensors measure signals from the target and make binary decisions according to the signals received. If the signal received by a sensor is greater than the threshold, the sensor makes decision 1, which means the sensor is fired. If the signal received by a sensor is less than the threshold, the sensor makes decision 0, which means the sensor is not fired.
Figure 1. Sensor field layout showing 441 binary sensors placed 9 m apart.

Based on decisions from sensors, the fusion center estimates the target position using MLE. In Figure 1, light circles denote unfired sensors and dark circles denote fired sensors. An asterisk denotes the true target position and a pentagram denotes the estimated target position.

**Target Localization Using Quantized Data**

Following the presentation in [29], a signal power model

\[ a_i^2 = \frac{G_i P_0}{(d_i/d_0)^\alpha} \]  

(1)

is considered. In (1), \( a_i \) is the signal amplitude at the \( i \)th sensor, \( G_i \) is the gain of the
ith sensor, and $P'_o$ is the power emitted by the target measured at a reference distance $d_o$. The Euclidean distance between the target and the ith sensor is

$$d_i = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2}$$

(2)

where $(x_i,y_i)$ and $(x_t,y_t)$ are the coordinates of sensor $i$ and the target, respectively.

The power decay exponent is $n$. It is assumed that $G_i = G$ for $i = 1,...,N$, such that $P_o = GP'_o$. The total number of sensors is $N$. Throughout this dissertation, $n = 2$ is used in (1) because experiments showed that the value of $n$ is very close to 2 for the acoustic signals that are widely used in many applications [28]. Moreover, it is assumed that the target is at least $d_o = 1$ meters away from any sensor at all times, resulting in the simplified signal power model

$$a_i^2 = \frac{P_o}{d_i^2}.$$ 

(3)

The measured signal at the ith sensor is modeled as

$$s_i = a_i + w_i$$

(4)

where $w_i$ is a Gaussian noise that follows the distribution $N(0,\sigma^2)$. At the ith sensor, the measured signal $s_i$ is quantized and transmitted to the fusion center.

The MLE approach for energy-based target localization using quantized data in [29] improved the previous work in [27][28] by using quantized data instead of analog data to transmit decisions from sensors to the fusion center. Figure 2 shows a model of target localization in WSNs using quantized data, as described in [29]. In this model, the output of the measurement process is $s_i$ and the output of the quantization process is $m_i$. 

5
Figure 2. Model of target localization in WSNs using quantized data, as described in [29]. Signal $s_i$ is the measured signal, $m_i$ is the decision made by the $i$th sensor, and $\hat{\theta}$ is the target location estimate.

The sensors send quantized multi-bit ($K$-bit) data to the fusion center. The data sent by sensors are $\mathbf{M} = \{m_i : i = 1, \ldots, N\}$, where $m_i$ can take any discrete value from 0 to $2^K - 1$. The set of quantization thresholds for the $i$th sensor is $\bar{\eta}_i = [\eta_{i0}, \eta_{i1}, \ldots, \eta_{iL}]$, where $\eta_{i0} = -\infty$, $\eta_{iL} = \infty$ and $L = 2^K$. Hence, the $i$th sensor supplies quantized data according to

$$m_i = \begin{cases} 0 & -\infty < s_i < \eta_{i1} \\ 1 & \eta_{i1} < s_i < \eta_{i2} \\ \vdots & \vdots \\ L-2 & \eta_{i(L-2)} < s_i < \eta_{i(L-1)} \\ L-1 & \eta_{i(L-1)} < s_i < \infty \end{cases}.$$  \hspace{1cm} (5)

The probability that $m_i$ takes specific value $m$ is

$$p(m_i = m|\theta) = Q\left(\frac{\eta_{il}-a_i}{\sigma}\right) - Q\left(\frac{\eta_{il+1}-a_i}{\sigma}\right) \quad (0 \leq m \leq L-1)$$  \hspace{1cm} (6)

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian distribution

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt.$$  \hspace{1cm} (7)
In [29], a log likelihood function corresponding to $m_i = m$ was developed based on (6). The overall log likelihood function was developed by adding the log likelihood function corresponding to decisions from every sensor. Then, $\theta$ was estimated by maximizing the overall log likelihood function. However, the MLE approach in [29] assumes that communication channels between sensors and the fusion center are perfect and that sensors will not fail. However, communication channels may not be perfect, and sensor faults may occur, which will distort the decision vector $M$ transmitted by sensors. Models of the imperfect communication channels are discussed, and a channel-aware MLE approach for energy-based target localization is presented in the next section. Using the framework of the channel-aware MLE approach for energy-based target localization, a fault-tolerant MLE approach for energy-based target localization will also be presented.

Channel-Aware Localization

In [30], a channel-aware energy-based target localization method was presented to mitigate performance degradation caused by communication channel imperfections. In this method, a model of an imperfect communication channel is incorporated into the MLE framework. Figure 3 shows a model of target localization using the channel-aware approach. The signal received at the sensor is measured, and the measurement $s_i$ is quantized. Because of imperfect communication channels, the input of the communication channel block is $m_i$, and the decision received by the fusion center is $\tilde{m}_i$ instead of $m_i$. The imperfect communication channel is characterized by $p(\tilde{m}_i|m_i)$ in [30]. By including appropriate transition probabilities,
the channel-aware energy-based target localization method can incorporate different types of communication channels. In [30], simulation results showed that if the communication channels were not perfect, the RMS estimation errors given by the channel-aware energy-based target localization method were lower than those given by the channel-unaware energy-based target localization method.

Figure 3. Model of target localization in WSNs using the channel-aware approach, as described in [30]. Signal $s_i$ is the measured signal, $m_i$ is the quantized signal, $\hat{m}_i$ is the decision received at the fusion center, and $\hat{\theta}$ is the target location estimate.

Formulation of the Channel-Aware MLE Approach for Energy-based Target Localization.

Since the channel-aware energy-based target localization method in [30] serves as the basis for other MLE approaches in this dissertation, the details of this method are reproduced here. The imperfect communication channels characterized by $p(\hat{m}_i|m_i)$ can distort the transmitted decision vector $\mathbf{M}$. The decision vector received at the fusion center is $\tilde{\mathbf{M}}=[\tilde{m}_1 \tilde{m}_2 ... \tilde{m}_N]^T$. The probability that $\tilde{m}_i$ is equal to $m$ is

$$p(\tilde{m}_i = m | \theta) = \sum_{m_i = 0}^{L-1} p(\tilde{m}_i = m | m_i)p(m_i | \theta).$$

(8)

For $\tilde{\mathbf{M}}=[\tilde{m}_1 \tilde{m}_2 ... \tilde{m}_N]^T$ received at the fusion center, the likelihood function is
The log-likelihood function is

$$p(\tilde{\mathbf{M}} | \theta) = \prod_{i=1}^{N} p(\tilde{m}_i | \theta) = \prod_{i=1}^{N} \left[ \sum_{m_i=0}^{l-1} p(\tilde{m}_i | m_i) p(m_i | \theta) \right].$$  \hspace{1cm} (9)

The log-likelihood function is

$$\ln p(\tilde{\mathbf{M}} | \theta) = \sum_{i=1}^{N} \ln \left[ \sum_{m_i=0}^{l-1} p(\tilde{m}_i | m_i) p(m_i | \theta) \right].$$ \hspace{1cm} (10)

The maximum likelihood estimator is

$$\hat{\theta} = \max_{\theta} \ln p(\tilde{\mathbf{M}} | \theta).$$ \hspace{1cm} (11)

If communication channels are perfect, \( p(\tilde{m}_i = m | m_i = m) = 1 \) and
\( p(\tilde{m}_i \neq m | m_i = m) = 0 \), which can be used to set \( p(\tilde{m}_i | m_i) \) in (8)-(11). For these settings, (8)-(11) form the MLE approach outlined in [29], which assumes that communication channels are perfect. Please note that in later chapters, the MLE approach in (8)-(11) was extended to incorporate models of other factors that may degrade localization performance.

**The Derivation of the CRLB.**

The estimation result of the MLE approach in (8)-(11) is unbiased. Therefore, RMS estimation errors are used as the performance criterion, and it is useful to derive Cramer-Rao lower bound (CRLB) on the variance of the estimation error. The CRLB for the channel-aware MLE approach for target localization in (8)-(11) is derived in [30] and is presented here for reference. If an unbiased estimate \( \hat{\theta} \) exists, the CRLB is
\[
E\{[\hat{\theta}(\text{M})-\theta][\hat{\theta}(\text{M})-\theta]^T\} \succeq \mathbf{J}^{-1}
\]  
(12)

where

\[
\mathbf{J} = -E\left[\nabla_{\theta} \nabla_{\theta}^T \ln p(\text{M} | \theta)\right]
\]  
(13)

is the Fisher information matrix (FIM).

First, \(\mathbf{J}(1,1)\) is derived:

\[
\frac{\partial^2 \ln p(\text{M} | \theta)}{\partial P_0^2} = \sum_{i=1}^{N} \sum_{l=1}^{L-1} \frac{\delta(\tilde{m}_i - l)}{p^2(\tilde{m}_i | \theta)} \left[ \frac{\partial \ln p(\tilde{m}_i | \theta)}{\partial P_0} \right]^2 + \frac{\delta(\tilde{m}_i - l) \delta^2 p(\tilde{m}_i | \theta)}{p(\tilde{m}_i | \theta) \partial P_0^2}.
\]  
(14)

Because the expectation of the second term of (14) is 0, the expectation of (14) is

\[
E\left[\frac{\partial^2 \ln p(\text{M} | \theta)}{\partial P_0^2}\right] = \sum_{i=1}^{N} \sum_{l=1}^{L-1} \frac{1}{p^2(\tilde{m}_i | \theta)} \left[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0} \right]^2.
\]  
(15)

In (15), \(p(\tilde{m}_i | \theta)\) is defined in (8). The derivative of \(p(\tilde{m}_i | \theta)\) with respect to \(P_0\) is

\[
\frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0} = \sum_{m_i=0}^{L-1} p(\tilde{m}_i | m_i) \frac{\partial p(m_i | \theta)}{\partial P_0}.
\]  
(16)

In (16), \(\frac{\partial p(m_i | \theta)}{\partial P_0}\) can be obtained as

\[
\frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0} = \frac{\partial}{\partial P_0} \left[ Q\left(\frac{\eta_{ii} - a_i}{\sigma}\right) - Q\left(\frac{\eta_{ii+1} - a_i}{\sigma}\right) \right] = \frac{d_i}{2\sqrt{2\pi} \sigma_0 \sqrt{P_0}} e^{-\frac{(\eta_{ii} - a_i)^2}{2\sigma_0^2}} - e^{-\frac{(\eta_{ii+1} - a_i)^2}{2\sigma_0^2}}.
\]  
(17)

Other elements of \(\mathbf{J}\) can be derived similarly.

**Fault-Tolerant Localization**

Sometimes, sensors will fail. Localization performance will be degraded if false decisions are received at the fusion center. In [31][32], a fault-tolerant MLE
approach for energy-based target localization is developed for binary sensor data by modeling the sensor faults and incorporating the sensor fault model into an MLE framework. Simulation results show that if sensor faults occurred, the RMS estimation errors given by the fault-tolerant localization method were lower than the RMS estimation errors given by the fault-intolerant localization method.

The fault-tolerant MLE approaches presented in [31] and [32] can be viewed as a particular example of the general framework in (8)-(11) if binary quantization is employed and if a sensor fault model is included (Figure 4). The fault-free decision made by the $i$th sensor is $m_i$. The decision received at the fusion center is $\tilde{m}_i$. Sensor faults are characterized by $p(\tilde{m}_i|m_i)$. Therefore, if $p(\tilde{m}_i|m_i)$ in (8)-(11) are set to the sensor fault probabilities, (8)-(11) can be used as a fault-tolerant MLE approach for energy-based target localization.

![Figure 4. Model of target localization in WSNs using sensor fault model, as described in [31][32]. Signal $s_i$ is the measured signal, $m_i$ is the quantized signal, $\tilde{m}_i$ is the decision received at the fusion center, and $\hat{\theta}$ is the target location estimate.](image)

Limitations of Current Work

The MLE approach for energy-based target localization in [29] uses quantized data but does not take into account imperfect communication channels or sensor faults. Imperfect communication channels are considered in [30]. A fault-tolerant
MLE approach using only binary data is presented in [31] and [32]. However, no work has considered simultaneous sensor faults and communication channel imperfections.

If the fusion center is located far away from sensors, the decisions may have to be relayed several times before reaching the fusion center. Communication channels in which decisions have to be relayed several times are called multi-hop communication channels. However, the MLE approach in [30] does not consider decisions transmitted through multi-hop communication channels.

In the work in [29] and [30], the fusion center localizes a target using decisions from sensors whose positions are accurately known. However, if the fusion center does not have accurate knowledge of sensor positions, localization performance suffers; sensor position errors can alter the probability in (9) because $a_i$ depends on $d_i$ according to the relation defined in (1). The fusion center may not have accurate sensor position information for several reasons. For example, sensor position errors may be caused by errors in the initial sensor layout process. These errors do not change over time. As another example, sensor position may drift randomly over time such that the values of sensor position errors vary over time.

Finally, energy conservation is important in WSNs where energy resources are limited. Saving energy may increase the lifetime of WSNs that use battery-powered sensors. Energy could be saved by allowing randomly selected sensors to sleep. However, sleeping sensors will not make detections and they will not send decisions to the fusion center; missing decisions from sensors will degrade localization performance. Therefore, energy is saved at the expense of localization
performance such that a tradeoff exists between saving energy and localization
performance. A sensor sleep model needs to be developed and integrated into the
MLE framework in (8)-(11). Although a multi-objective optimization method has
been used for the target detection problem in [33], to the best of our knowledge, no
one has used a multi-objective optimization method to balance localization
performance and energy consumption for energy-based target localization.

Objectives and Specific Aims

This dissertation has three objectives. This first objective is to improve the
performance of the energy-based target localization method in [29] by addressing
some factors that degrade localization performance. Multi-hop communication was
modeled as a single equivalent BSC model and a direct method was developed for
determining the coefficients in this equivalent BSC model. The BSC model in [30]
was replaced by the equivalent BSC model to allow the MLE approach in [30] to be
used when decisions are transmitted over multi-hop communication channels.
Models of sensor faults and communication channel imperfections were incorporated
into the MLE framework in (8)-(11) to improve localization performance if faulty
sensor and imperfect communication channels occur at the same time. Another factor
is sensor position uncertainty. As mentioned earlier, sensor position uncertainty will
degrade localization performance of the energy-based target localization method in
[29]. Random sensor position errors were modeled, and the model was incorporated
into the likelihood function to develop a robust MLE method that alleviated
localization performance degradation caused by sensor position uncertainty.
The second objective of this dissertation is to use a sensor sleep scheme to save energy and develop an MLE approach for target localization for networks that include sensors that are allowed to sleep at random. First, a model was developed for using randomly sleeping sensors to save energy and the sleep probabilities in this decision transition model were determined. Then, values of \( p(\tilde{m}_i|m_i) \) in (8)-(11) were set to the sleep probabilities to allow (8)-(11) to be used as a sleep-accommodating MLE approach for target localization.

In achieving the objectives, an energy-adjustable, channel-aware, and fault-tolerant MLE approach for target localization was developed. Figure 5 shows a detailed model of this approach. The sensor measures the signal received and the measurement \( s_i \) is quantized. The quantized measurement is \( m_i \). Because of the sensor sleep scheme, the output of the sensor sleep block is \( \tilde{m}_i \) instead of \( m_i \). Similarly, the output of the sensor fault block is \( \bar{m}_i \) and the decision received at the fusion center is \( \tilde{m}_i \).

The third objective of this dissertation is to use a multi-objective optimization method to balance energy utilization and localization performance for the energy-adjustable MLE approach. Models were developed to describe energy utilization and localization performance in WSNs. Then, localization performance and energy utilization were balanced using a multi-objective optimization method.

The rest of this dissertation is organized as follows. A MLE approach for energy-based target localization using decisions transmitted over multi-hop communication channels is presented in Chapter II. In Chapter III, sensor position uncertainty is considered. Three different types of sensor position uncertainty were
modeled, and the models were included in the MLE framework. In Chapter IV, an energy-adjustable, fault-tolerant, and channel-aware target localization method is presented. Finally, in Chapter V, a multi-objective optimization method is used to balance energy consumption and localization performance for energy-based target localization method using both one-dimensional and two-dimensional sensor arrays.
Figure 5. Model of target localization in WSNs using the communication channel model, the sensor fault model, and the sensor sleep model. Signal $s_i$ is the measured signal, $m_i$ is the quantized signal, $\hat{m}_i$ is the output of sensor sleep block, $\overline{m}_i$ is the output of fault block, $\tilde{m}_i$ is the decision received at the fusion center, and $\hat{\theta}$ is the target location estimate.
II. TARGET LOCALIZATION USING DECISIONS TRANSMITTED OVER MULTI-HOP COMMUNICATION CHANNELS

In the MLE approach for energy-based target localization using quantized data presented in [29], the fusion center uses decisions from sensors to localize a target. The approach assumes perfect communication channels between sensors and the fusion center. The work in [30] improves the MLE approach for energy-based target localization using quantized data by modeling imperfect one-hop communication channels and incorporating the communication channel model into the MLE framework. However, if the fusion center is located far away from sensors, decisions made by sensors may have to be relayed several times before reaching the fusion center. Communication channels in which decisions have to be relayed several times are called multi-hop communication channels. In this section, the work in [30] is extended for decisions transmitted over multi-hop communication channels. The main content of this section has been published in [34].

For distributed detection, decision fusion rules based on decisions transmitted through multi-hop WSNs are presented in [35], in which the communication channels between two relay nodes were modeled as a BSC under some specified conditions. In [35], a recursive method is used to determine the coefficients in this BSC model.
However, this recursive method is complex and computationally intensive.

In this chapter, the multi-hop transmission scheme between the sensor and the fusion center in WSNs was modeled as a single equivalent BSC model. In addition, a direct method that is simple to use in finding the transmission coefficients in this equivalent BSC model was developed.

Finally, target localization was performed using the channel-aware MLE approach in (8)-(11), where values for $p(\tilde{m}_i|m_i)$ were set to the transition probabilities in the equivalent BSC model for multi-hop transmission. This approach is the same as using the channel-aware MLE method presented in [30], with the BSC model coefficients set to the coefficients of the equivalent BSC model for multi-hop transmission. Simulation results for target localization based on decisions transmitted over multi-hop communication channels showed that RMS estimation errors were close to the CRLB.

Localization Using Decisions Transmitted Over a BSC

Following the derivation in [29], the signal decay model (3) is used. The received signal at sensor $i$ is corrupted by a Gaussian noise, as in (4). After receiving the signal $s_i$, the sensor $i$ quantizes the signal $s_i$ to $m_i$ according the threshold $\eta_i$.

Under some assumptions, such as phase-coherent reception and identical channel characteristics, each hop can be modeled as a BSC using the binary decisions $\{-1,1\}$ [35]. In this chapter, the binary decisions $\{-1,1\}$ are used. The quantization process is
The probability that transmitted decision $m_i$ takes value $m$ is

$$p(m_i = m|\theta) = \begin{cases} 1 - Q(\frac{\eta_i - a_i}{\sigma}) & (m = -1) \\ Q(\frac{\eta_i - a_i}{\sigma}) & (m = 1) \end{cases}$$

The decision transmitted through a BSC is $m_i$ and the received decision is $\tilde{m}_i$. The transition relation between $m_i$ and $\tilde{m}_i$ is modeled as a BSC (Figure 6). The BSC is characterized by the crossover probability $p$ and the probability of correct transmission $q$.

![Figure 6. Binary symmetric channel.](image)

Next, the BSC model was incorporated into the MLE framework in (8)-(11) using decisions $\{-1, 1\}$. The probability that $\tilde{m}_i$ takes the value $m$ is given by

$$p(\tilde{m}_i = m|\theta) = \sum_{m_i \in \{-1, 1\}} p(\tilde{m}_i = m|m_i)p(m_i|\theta)$$
where \( p(m_i | \theta) \) is defined in (19), and \( p(\tilde{m}_i = m_i | \cdot) \) indicates the transition probabilities in the BSC model in Figure 6. For a decision vector \( \mathbf{M} = [\tilde{m}_1, \tilde{m}_2, ..., \tilde{m}_N]^T \) received at the fusion center, the fusion center estimates \( \theta = [P_0, x_i, y_i]^T \) by finding \( \theta \) to maximize

\[
\ln p(\mathbf{M} | \theta) = \sum_{i=1}^{N} \ln \left( \sum_{m_i \in \{-1,1\}} p(\tilde{m}_i = m_i | m_i)p(m_i | \theta) \right). \tag{21}
\]

The maximum likelihood estimator is

\[
\hat{\theta} = \max_{\theta} \ln p(\mathbf{M} | \theta). \tag{22}
\]

If an unbiased estimate \( \hat{\theta} \) exists, the CRLB of the estimate is

\[
E\{[\hat{\theta}(\mathbf{M}) - \theta][\hat{\theta}(\mathbf{M}) - \theta]^T\} \geq J^{-1}. \tag{23}
\]

where

\[
J = -E\left[ \nabla_\theta \nabla_\theta^T \ln p(\mathbf{M} | \theta) \right]. \tag{24}
\]

Details of the derivation of \( J \) used to compute the CRLB are presented in [30].

The Equivalent BSC Model for Multi-Hop Transmission

The method described in the previous section can be used only for one-hop WSNs. However, in multi-hop WSNs, decisions have to be relayed several times before reaching the fusion center. If all hops are identical, the multi-hop communication scheme can be modeled using an equivalent BSC model, as shown in Figure 7. The equivalent BSC
model in Figure 7 can replace the BSC model in Figure 6 if \( p \) and \( q \) in the BSC model in Figure 6 are set to \( p_{BSC} \) and \( q_{BSC} \), respectively. Then, the MLE framework in (21)-(24) is used as the target localization method for decisions transmitted over multi-hop communication channels. In the following, a new direct method is developed to determine the coefficients of the equivalent BSC model.

![Figure 7. Equivalent BSC model for multi-hop transmission: a) Multi-hop model, b) Equivalent multi-hop model.](image)

**Direct Method to Determine Coefficients for the Equivalent BSC Model for Multi-Hop Transmission**

A direct method based on the binomial theorem is used to determine the transmission coefficients in the equivalent BSC model. Mathematical induction is used to prove the direct method. Theoretical results are compared with results obtained from inspection of Figure 7.

The equivalent BSC model is described using the crossover probability \( p_{BSC} \) and
probability of correct transmission $q_{BSC}$, which are defined as

$$p(\hat{m}_i = 1 | m_i = 1) = p(\hat{m}_i = -1 | m_i = -1) = q_{BSC} = 1 - p_{BSC}$$ (25)

$$p(\hat{m}_i = 1 | m_i = -1) = p(\hat{m}_i = -1 | m_i = 1) = p_{BSC}.$$ (26)

Theorem: In a multi-hop WSN with $n$ hops, it is assumed that every hop has the same crossover probability $p$ and the same probability of correct transmission $q$.

From the binomial theorem in [36], the algebraic expansion of $(q + p)^n$ is sorted and written in descending order of $q$

$$(q + p)^n = \sum_{k=0}^{n} \binom{n}{k} q^{n-k} p^k$$ (27)

$$= q^n + C_n^1 q^{n-1} p + C_n^2 q^{n-2} p^2 + C_n^3 q^{n-3} p^3 + \cdots + C_n^{n-1} q p^{n-1} + p^n.$$

The value of $p_{BSC}$ is equal to the sum of all of the even numbered terms

$$\{ C_n^1 q^{n-1} p, C_n^3 q^{n-3} p^3, \cdots \}$$ in (27). The value of $q_{BSC}$ is equal to the sum of all of the odd numbered terms

$$\{ q^n, C_n^2 q^{n-2} p^2, \cdots \}$$ in (27).

Proof: Basis: for a single hop model, from inspection of Figure 7, the coefficients of the equivalent BSC model can be expressed as

$$p(\hat{m}_i = 1 | m_i = 1) = p(\hat{m}_i = -1 | m_i = -1) = q$$ (28)

$$p(\hat{m}_i = 1 | m_i = -1) = p(\hat{m}_i = -1 | m_i = 1) = (1 - q) = p.$$ (29)

The expansion of $(q + p)^1$ can be denoted as $(q + p)^1 = q + p$. Thus, $q_{BSC}^1 = q$ (the sum of all odd numbered terms in the expansion of $(q + p)^1$) and $p_{BSC}^1 = p$ (the sum of all even numbered terms in the expansion of $(q + p)^1$). Therefore, for $n = 1$, the results
(28) and (29) from our theory are the same as those from inspection of Figure 7.

If \( n = 2 \), from inspection of Figure 7, the coefficients of the corresponding BSC model are

\[
p(\tilde{m}_i = 1 | m_i = 1) = p(\tilde{m}_i = -1 | m_i = -1) = q^2 + p^2
\]

\[
p(\tilde{m}_i = -1 | m_i = 1) = p(\tilde{m}_i = 1 | m_i = -1) = (1 - q) = 2pq.
\]

For the multi-hop transmission scheme using \( n \) hops, \( q^{n}_{BSC} \) denotes the probability of correct transmission, and \( p^{n}_{BSC} \) denotes the crossover probability for the equivalent BSC model. Using (27), the expansion of \((q + p)^2\) can be expressed as

\[(q + p)^2 = q^2 + 2qp + p^2.\]

The coefficients are

\[
p^{n}_{BSC} = 2qp \quad \text{(the sum of all even numbered terms in the expansion of \((q + p)^2\))}
\]

and

\[
q^{n}_{BSC} = q^2 + p^2 \quad \text{(the sum of all odd numbered terms in the expansion of \((q + p)^2\)).}
\]

Therefore, our theory holds for \( n = 2 \).

Induction steps:

a) Assume our theory holds for \( n \) hops. From (27), the following results are obtained.

1) If \( n \) is an odd number, then \((q + p)^n\) has an even number of terms, and

\[
q^{n}_{BSC} = q^n + C_n^2 q^{n-2} p^2 + \cdots + C_n^{n-3} q^3 p^{n-3} + C_n^{n-1} q p^{n-1}
\]

\[
p^{n}_{BSC} = C_n^1 q^{n-1} p + C_n^3 q^{n-3} p^3 + \cdots + C_n^{n-2} q^2 p^{n-2} + p^n.
\]

2) If \( n \) is an even number, \((q + p)^n\) has an odd number of terms, and

\[
q^{n}_{BSC} = q^n + C_n^2 q^{n-2} p^2 + \cdots + C_n^{n-2} q^2 p^{n-2} + p^n
\]

\[
p^{n}_{BSC} = C_n^1 q^{n-1} p + C_n^3 q^{n-3} p^3 + \cdots + C_n^{n-3} q^3 p^{n-3} + C_n^{n-1} q p^{n-1}.
\]
b) If the number of hops is \( n+1 \), the transmission coefficients in the equivalent BSC model are

\[
(q + p)^{n+1} = (q + p)^n (q + p) \\
= q^{n+1} + C_n^1 q^n p + C_n^2 q^{n-1} p^2 + C_n^3 q^{n-2} p^3 + \cdots \\
+ C_n^{n-1} q^{n} p^{n-1} + q^n p + C_n^{1} q^{n+1} p^2 + \\
C_n^{2} q^{n-2} p^3 + C_n^{3} q^{n-3} p^4 + \cdots + C_n^{n-1} q^{n} p^{n+1} \\
= q^{n+1} + (C_n^{1} + 1) q^{n} p + (C_n^{2} + C_n^{1}) q^{n-1} p^2 + \\
(C_n^{3} + C_n^{2}) q^{n-2} p^3 + \cdots + (C_n^{n-1} + 1) q^{n} p^{n+1}.
\]

Applying the theory to (36), the following results are obtained.

1) If \( n + 1 \) is an odd number, then \((q + p)^{n+1}\) will have an even number of terms, and

\[
q_{BSC}^{n+1} = q^{n+1} + (C_n^{2} + C_n^{1}) q^{n-1} p^2 + \cdots + (C_n^{n-1} + 1) q^n p^n \\
= (C_n^{1} + 1) q^n p + (C_n^{2} + C_n^{1}) q^{n-1} p^2 + \cdots + p^{n+1}.
\]

2) If \( n + 1 \) is an even number, then \((q + p)^{n+1}\) will have an odd number of terms, and

\[
q_{BSC}^{n+1} = q^{n+1} + (C_n^{n} + C_n^{n-1}) q^{n-2} p^3 + \cdots + p^{n+1} \\
p_{BSC}^{n+1} = (C_n^{1} + 1) q^n p + (C_n^{2} + C_n^{1}) q^{n-2} p^3 + \cdots + (C_n^{n-1} + 1) q^n p^n.
\]

By inspection of Figure 7 a),

\[
q_{BSC}^{n+1} = q_{BSC}^n \times q + p_{BSC}^n \times p \\
q_{BSC}^{n+1} = q_{BSC}^n \times p + p_{BSC}^n \times q
\]

1) If \( n \) is an even number, then \( n + 1 \) is an odd number, and \((q + p)^n\) has an odd number of terms. From Figure 7,
This result from inspection of Figure 7 validates the theoretical results (37) and (38).

2) If \( n \) is an odd number, then \( n+1 \) is an even number, and \((q+p)^n\) has an even number of terms. By inspection of Figure 7 a),

\[
q^{n+1}_{\text{BSC}} = (q^n + C_n^2 q^{n-2} p^2 + \cdots + C_n^{n-2} q^2 p^{n-2} + p^n) \times q +
\]

\[
(C_n^n q^{n-1} p + C_n^3 q^{n-3} p^3 + \cdots + C_n^n q^1 p^{n-1}) \times p
\]

\[
= q^{n+1} + (C_n^2 + C_n^1) q^{n-1} p^2 + \cdots + (C_n^{n-2} + C_n^{n-3}) q^2 p^{n-2} + (1 + C_n^{n-1}) q p^n
\]

\[
p^{n+1}_{\text{BSC}} = (q^n + C_n^2 q^{n-2} p^2 + \cdots + C_n^{n-2} q^2 p^{n-2} + p^n) \times p
\]

\[
+(C_n^n q^{n-1} p + C_n^3 q^{n-3} p^3 + \cdots + C_n^n q^1 p^{n-1}) \times q
\]

\[
= (1 + C_n^1) q^n p + (C_n^2 + C_n^3) q^{n-2} p^3 + \cdots + (C_n^{n-2} + C_n^{n-3}) q^2 p^{n-2} + p^{n+1}
\]

The theoretical results (39) and (40) are identical to the results (45) and (46) derived from inspection of the transition diagram. Therefore, the results from our theory are the same as the results from inspection of the multi-hop transition diagram.

Simulation Setup

Because \( \eta \) is very important for the MLE approach for energy-based target localization using decisions transmitted over multi-hop communication channels, an appropriate value of \( \eta \) has to be determined for use in the simulations where RMS
estimation errors are compared to the CRLB. If \( \eta \) is too high, too few sensors fire, and the CRLB is not valid. If \( \eta \) is too low, too many sensors fire, the sensor field is saturated, and the CRLB is also not valid. Therefore, it is important to select an appropriate value of \( \eta \) for which the CRLB is valid. The appropriate value of \( \eta \) was determined in Monte Carlo simulations by comparing the average of the normalized estimation error squared (NEES) values generated for different \( \eta \) values with the 95% confidence region for the average NEES value. As described in [29], the NEES is defined by \( \varepsilon_\theta = (\theta - \hat{\theta})^T J (\theta - \hat{\theta}) \). If the number of Monte Carlo simulations used is \( N_m \), the average NEES is \( \bar{\varepsilon}_\theta = \frac{1}{N_m} \sum_{i=1}^{N_m} \varepsilon_{\theta}^i \).

The average NEES values used to determine the appropriate value of \( \eta \) were generated using 1000 Monte Carlo simulations in which targets were placed according to a random uniform distribution within a circle having radius equal to 25 and center at (12, 13) (Figure 8). The 90% confidence region for the average NEES value corresponding to 1000 Monte Carlo simulations and estimation of three parameters is [2.8737 3.1285]. The average NEES value corresponding to \( \eta = 3 \) was within the confidence region and the average NEES values were outside the confidence region for other values of \( \eta \) (Table 1). Therefore, \( \eta = 3 \) was used for the simulations. Since the average NEES values in Table 1 were generated using random target positions, the MLE approach for energy-based target localization using decisions transmitted over multi-hop communication channels is valid for different target positions.
Figure 8. Sensor field with random target locations.

TABLE 1

<table>
<thead>
<tr>
<th>Threshold, $\eta$</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2213</td>
</tr>
<tr>
<td>2</td>
<td>3.1521</td>
</tr>
<tr>
<td>3</td>
<td>3.1245</td>
</tr>
<tr>
<td>4</td>
<td>3.1498</td>
</tr>
</tbody>
</table>

We set different $q$ values and calculated $p_{BSC}$ and $q_{BSC}$ in the equivalent BSC model using (32)-(35) to see the effect of $q$ on $p_{BSC}$ and $q_{BSC}$. Simulations were also carried out to demonstrate the effect of the number of hops on the RMS estimation errors for the estimates computed using (22) and on the CRLB computed using (23). In these simulations, $\eta = 3$, $P_0 = 10000$, and $(x_t, y_t) = (12,13)$. The crossover probability, $p$,
was set to 0.01 and then to 0.05. The number of hops, $n$, was varied from 1 to 8. The RMS estimation errors were calculated based on 1000 Monte Carlo runs.

Results and Discussion

The effect of $q$ and $n$ on $p_{BSC}$ and $q_{BSC}$ values was examined. The value of $q_{BSC}$ decreased as $n$ increased (Tables 2, 3, and 4). It is expected that if $n$ is large enough, $q_{BSC}$ will be less than 0.5, and the communication channel will become unsuitable to transmit any decision.

<table>
<thead>
<tr>
<th>Number of hops, $n$</th>
<th>Probability of correct transmission, $q_{BSC}$</th>
<th>Crossover probability, $p_{BSC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9500</td>
<td>0.0500</td>
</tr>
<tr>
<td>2</td>
<td>0.9050</td>
<td>0.0950</td>
</tr>
<tr>
<td>3</td>
<td>0.8645</td>
<td>0.1355</td>
</tr>
<tr>
<td>4</td>
<td>0.8280</td>
<td>0.1720</td>
</tr>
<tr>
<td>5</td>
<td>0.7952</td>
<td>0.2048</td>
</tr>
<tr>
<td>6</td>
<td>0.7657</td>
<td>0.2343</td>
</tr>
<tr>
<td>7</td>
<td>0.7391</td>
<td>0.2609</td>
</tr>
<tr>
<td>8</td>
<td>0.7152</td>
<td>0.2848</td>
</tr>
</tbody>
</table>

Results showed that the CRLB and RMS estimation errors increased as $n$ increased (Figures 9 and 10). When $n$ increased, the RMS estimation errors deviated from the CRLB. The RMS estimation errors were closer to the CRLB for $p = 0.01$. 

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Furthermore, the RMS estimation errors were close to the CLRB when $n$ was low.

Localization performance worsened when $p$ and $n$ were such that $q_{BSC}$ was low.

**TABLE 3**
CROSSOVER PROBABILITY AND THE PROBABILITY OF CORRECT TRANSMISSION IN THE EQUIVALENT BSC MODEL ($q = 0.98$)

<table>
<thead>
<tr>
<th>Number of hops, $n$</th>
<th>Probability of correct transmission, $q_{BSC}$</th>
<th>Crossover probability, $p_{BSC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9800</td>
<td>0.0200</td>
</tr>
<tr>
<td>2</td>
<td>0.9608</td>
<td>0.0392</td>
</tr>
<tr>
<td>3</td>
<td>0.9424</td>
<td>0.0576</td>
</tr>
<tr>
<td>4</td>
<td>0.9247</td>
<td>0.0753</td>
</tr>
<tr>
<td>5</td>
<td>0.9077</td>
<td>0.0923</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>8</td>
<td>0.8607</td>
<td>0.1393</td>
</tr>
</tbody>
</table>

**TABLE 4**
CROSSOVER PROBABILITY AND THE PROBABILITY OF CORRECT TRANSMISSION IN THE EQUIVALENT BSC MODEL ($q = 0.99$)

<table>
<thead>
<tr>
<th>Number of hops, $n$</th>
<th>Probability of correct transmission, $q_{BSC}$</th>
<th>Crossover probability, $p_{BSC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>3</td>
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</tr>
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<td>4</td>
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</tr>
<tr>
<td>5</td>
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<td>6</td>
<td>0.9429</td>
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</tr>
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<td>7</td>
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<td>0.0659</td>
</tr>
<tr>
<td>8</td>
<td>0.9254</td>
<td>0.0746</td>
</tr>
</tbody>
</table>

Future work could consider other types of communication channels, such as
widely-used Rayleigh fading channel models. Moreover, the relation between $p_{BSC}$ and $n$ is needed to predict the upper limit of $n$ for suitable transmission of decisions. Ideally, the relation should be expressed in a closed-form formula.

![Figure 9. RMS estimation errors and CRLB ($p = 0.01$, $(x_t, y_t) = (12,13)$ and $\eta = 3$).](image)

**Summary**

In this chapter, the work in [30] was extended for decisions transmitted over multi-hop communication channels. Moreover, a direct method to calculate the coefficients in the equivalent BSC model was devised. The direct method to calculate coefficients was proved using mathematical induction. The direct method is easier to use than the recursive method in [35].
Figure 10. RMS estimation errors and CRLB ($p = 0.05$, $(x_t, y_t) = (12,13)$ and $\eta = 3$).
III. LOCALIZATION BASED ON MODELS OF SENSOR POSITION UNCERTAINTY

In the MLE approach for energy-based target localization in [29], the fusion center estimates target power and target location using sensor decisions and knowledge of the sensor locations. It is assumed that the fusion center has accurate information about sensor locations. However, sensor location information may not be known accurately at the fusion center due to initial sensor layout errors or due to sensor movement. If the fusion center localizes the target using imperfect sensor location information, localization performance suffers. In this chapter, sensor position uncertainty is modeled, and the model is incorporated into the MLE framework in [29]. The main content of this chapter has been published in [37].

The first model of sensor position uncertainty describes a line position error in which sensors are located along a line passing through the target and the assumed sensor position. This model is called the line-position error model (LEM). The second model, called the time-varying circle-position error model (TVCEM), describes a type of sensor position uncertainty in which sensors drift from time to time within the circle centered at the assumed sensor positions. The third model, called the circle error model (CEM), describes sensor position uncertainty caused by the initial layout of sensors. In the CEM,
sensors are randomly located within a circle centered at the assumed sensor position. However, sensors do not move after initial deployment.

Simulation results presented in this chapter show that if sensor position errors were described by the LEM, RMS localization errors given by the LEM MLE approach were lower than those given by the MLE approach in [29]. Moreover, simulation showed that the LEM MLE approach could also be used as an approximation approach to alleviate performance degradation caused by the sensor position errors described by the TVCEM and the CEM.

Localization Based on the LEM

According to (3), sensor position errors will affect the value of \( d_i \) and, therefore, the errors will impact the value of \( a_i \). If the fusion center estimates the target position based on inaccurate knowledge of sensor positions, localization performance will suffer. Our solution is to model sensor position errors and incorporate the model into the MLE framework.

In the LEM representation (Figure 11), sensors are located along the line passing through the assumed sensor position and the target location. The distance between the target and the assumed sensor position is denoted as \( d_i \). The distance between the assumed sensor position and the actual sensor position is denoted as \( h_i \), which is a uniformly distributed random distance error. The distance between the actual position of
the \( i \)th sensor and the target can be modeled as \( d_i + h_i \). The signal decay model in (3) is modified to account for \( h_i \) by writing

\[
 a_i^2 = \frac{P_0}{(d_i + h_i)^2}. \quad h_i \in U[a,b]
\]

(47)

Because of the presence of noise, the received signal can be expressed as

\[
s_i = a_i + w_i
\]

(48)

where \( w_i \) is a noise with Gaussian distribution \( N(0, \sigma^2) \).

Figure 11. The LEM representation.

Using the knowledge of the distribution of \( h_i \), the probability density function (PDF) of \( a_i \) can be expressed as

\[
f(a_i) = \frac{\sqrt{P_0}}{a_i^2} f(h_i) = \frac{\sqrt{P_0}}{a_i^2} \frac{1}{b-a} a_i \in \left[ \frac{\sqrt{P_0}}{d_i + b}, \frac{\sqrt{P_0}}{d_i + a} \right].
\]

(49)

According to [29] and [36], the PDF of the sum of two independent random variables is equal to the convolution of their PDFs. Therefore, the PDF of \( f(s_i) \) can be expressed as

\[
f(s_i) = \int_{d_i+b}^{\sqrt{P_0}/d_i+a} f_{w_i}(\tau) f_{s_i}(s_i - \tau) = \int_{d_i+b}^{\sqrt{P_0}/d_i+a} \sqrt{P_0} e^{-\frac{(s_i-\tau)^2}{2\sigma^2}} \cdot \frac{1}{2\pi \sigma} e^{-\frac{\tau^2}{2\sigma^2}} d\tau
\]

\[
= \frac{\sqrt{P_0}}{\sqrt{2\pi \sigma} \sigma(b-a)} \int_{d_i+b}^{\sqrt{P_0}/d_i+a} e^{-\frac{(s_i-\tau)^2}{2\sigma^2}} \tau \cdot \frac{1}{\tau^2} d\tau.
\]

(50)
After receiving the signals from the target, the \( i \)th sensor will quantize the received signal \( s_i \) to a decision \( m_i \) according to a set of pre-determined thresholds

\[
\eta_i = [\eta_{i0}, \eta_{i1}, \ldots, \eta_{iL}].
\]  

(51)

In (51), \( \eta_{i0} = -\infty \), and \( \eta_{iL} = \infty \). The quantization process is denoted by

\[
m_i = \begin{cases} 
0 & -\infty < s_i < \eta_{i1} \\
1 & \eta_{i1} < s_i < \eta_{i2} \\
\vdots & \vdots \\
L-2 & \eta_{i(L-2)} < s_i < \eta_{i(L-1)} \\
L-1 & \eta_{i(L-1)} < s_i < \infty
\end{cases}.
\]  

(52)

For a given \( \theta = [P_0, x_i, y_i]^T \), the probability that \( m_i \) takes the value \( l \) is given by

\[
p_d(\eta, \theta) = R(\eta_{l_0}) - R(\eta_{l+1}) = \int_{\eta_{l}}^{\eta_{l+1}} f(s_i)ds_i \quad (0 \leq l \leq L-1),
\]  

(53)

in which \( R(x) \) is

\[
R(x) = \int_{x}^{\infty} f(s_i)ds_i.
\]  

(54)

For a particular decision vector received

\[
M = [m_1, m_2, \ldots, m_{N-1}, m_N],
\]  

(55)

the fusion center maximizes

\[
\ln p(M|\theta) = \sum_{i=1}^{N-1} \sum_{l=0}^{L-1} \delta(m_i - l) \ln p_d(\eta, \theta)
\]  

(56)

to estimate \( \theta = [P_0, x_i, y_i]^T \). In (56), \( \delta(x) \) is

\[
\delta(x) = \begin{cases} 
1, & x = 0 \\
0, & x = 0
\end{cases}.
\]  

(57)

The maximum likelihood estimator is

\[
\hat{\theta} = \max_{\theta} \ln p(M|\theta).
\]  

(58)
If an unbiased estimator $\hat{\theta}$ exists, the CRLB is

$$E\{[\hat{\theta}(\mathbf{M}) - \theta][\hat{\theta}(\mathbf{M}) - \theta]^T\} \geq \mathbf{J}^{-1}$$

where

$$\mathbf{J} = -E[\nabla_{\theta} \nabla_{\theta}^T \ln p(\mathbf{M}|\theta)].$$

The details of the method to calculate CRLB for the LEM MLE approach are presented in following subsections.

The Derivation of the CRLB

The elements of $\mathbf{J}$ for the log-likelihood function (56) can be derived using a method similar to that employed in [29].

1) First $\mathbf{J}(1,1)$ is derived:

$$\frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial \mathbf{P}_0^2} = \sum_i \sum_i \frac{\delta(m_i - l)}{p_a^2(\eta_i, \theta)} \left[ \frac{\partial \ln p_a(\eta_i, \theta)}{\partial \mathbf{P}_0} \right]^2 + \frac{\delta(m_i - l)}{p_a(\eta_i, \theta)} \frac{\partial^2 p_a(\eta_i, \theta)}{\partial \mathbf{P}_0^2}. \quad (61)$$

The expectation of (61) is

$$E\left[\frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial \mathbf{P}_0^2}\right] = \sum_i \sum_i \frac{1}{p_a^2} \left[ \frac{\partial \ln p_a}{\partial \mathbf{P}_0} \right]^2. \quad (62)$$

To derive (62), $E[\delta(m_i - l)] = p_a(\eta_i, \theta)$ has been used. In (62), $p_a$ is

$$p_a(\eta_i, \theta) = \frac{\eta_i}{\sqrt{2\pi\sigma(b-a)}} \int_{\eta_i}^{\eta_i + a} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(s-x)^2}{2\sigma^2}} ds. \quad (63)$$

The derivative of (63) with respect to $\mathbf{P}_0$ is

$$\frac{\partial p_a(\eta_i, \theta)}{\partial \mathbf{P}_0} = Q\left(\frac{\eta_i - a_i}{\sigma}\right) - Q\left(\frac{\eta_i + a_i}{\sigma}\right). \quad (64)$$
\[ \Delta_1 = \int_{\eta_i}^{\eta_{i+1}} \left[ \frac{1}{2\sqrt{2\pi}\sigma(b-a)} \int_{\frac{\sqrt{P_0}}{d_i+a}}^{\frac{\sqrt{P_0}}{d_i+b}} \frac{1}{\tau^2} e^{-\frac{(\tau - \tau')^2}{2\sigma^2}} d\tau \\
+ \frac{(d_i + a)e^{\frac{(d_i - \sqrt{P_0})^2}{2\sigma^2}} - (d_i + b)e^{\frac{(-d_i + \sqrt{P_0})^2}{2\sigma^2}}}{2\sqrt{2\pi}\sigma(b-a)P_0} \right]^2 \, ds_i \]  

, (65)

then

\[ E \left[ \frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial P_0^2} \right] = \sum_i \sum_l -\frac{\Delta_1}{p_{il}}. \]  

(66)

2) \( J(2,2) \) element

In (53), \( p_{il}(\eta_i, \theta) \) is

\[ p_{il}(\eta_i, \theta) = \int_{\eta_i}^{\eta_{i+1}} \sqrt{P_0} \sqrt{2\pi}\sigma(b-a) \int_{\frac{\sqrt{P_0}}{d_i+a}}^{\frac{\sqrt{P_0}}{d_i+b}} \frac{1}{\tau^2} e^{-\frac{(\tau - \tau')^2}{2\sigma^2}} d\tau ds_i \quad (0 \leq l \leq L - 1). \]  

(67)

The partial derivative of (67) with respect to \( x_i \) is

\[ \frac{\partial p_{il}(\eta_i, \theta)}{\partial x_i} = \int_{\eta_i}^{\eta_{i+1}} \sqrt{P_0} \sqrt{2\pi}\sigma(b-a) \left[ \frac{e^{\frac{(\tau - \sqrt{P_0})^2}{2\sigma^2}} (d_i + a)^2 P_0 \frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_0}}{d_i + a} \right)}{P_0 d_i (d_i + a)} - e^{\frac{(\tau - \sqrt{P_0})^2}{2\sigma^2}} (d_i + b)^2 P_0 \frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_0}}{d_i + b} \right) \right] ds_i \]

\[ = \int_{\eta_i}^{\eta_{i+1}} \sqrt{P_0} \sqrt{2\pi}\sigma(b-a) \left[ \frac{e^{\frac{(\tau - \sqrt{P_0})^2}{2\sigma^2}} (d_i + a)^2 P_0 (x_i - x_i) - \sqrt{P_0} (x_i - x_i)}{P_0 d_i (d_i + a)^2} \right] ds_i \]

\[ = \int_{\eta_i}^{\eta_{i+1}} \sqrt{P_0} \sqrt{2\pi}\sigma(b-a) \left[ \frac{e^{\frac{(\tau - \sqrt{P_0})^2}{2\sigma^2}} (d_i + a)^2 P_0 (x_i - x_i) - \sqrt{P_0} (x_i - x_i)}{d_i \sqrt{P_0} (x_i - x_i)} \right] ds_i. \]  

(68)
The equation

$$\frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_0}}{d_i + a} \right)$$

$$= \frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_0}}{\sqrt{(x_i - x_i)^2 + (y_i - y_i)^2 + a}} \right)$$

$$= \frac{-\sqrt{P_0}}{(d_i + a)^2} \frac{1}{d_i} \frac{1}{2} 2(x_i - x_i)$$

$$= \frac{-\sqrt{P_0}}{d_i (d_i + a)^2}$$

is used to derive (68). If

$$\Delta_2 = \left( \int_{\eta_i (\tau_i)}^{\eta_i (\tau_i + 1)} \sqrt{P_0} \left( \frac{e^{-\frac{(x_i - \sqrt{P_0} b)^2}{2\sigma^2}}}{d_i \sqrt{P_0}} - e^{-\frac{(x_i - \sqrt{P_0} a)^2}{2\sigma^2}} \frac{1}{d_i \sqrt{P_0}} \right) ds_i \right)^2,$$

then

$$E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial x_i^2} \right] = \sum_i \sum_l -\frac{\Delta_2}{p_{il}}.$$

3) J(3,3) element

If

$$\Delta_3 = \left( \int_{\eta_i (\tau_i)}^{\eta_i (\tau_i + 1)} \sqrt{P_0} \left( \frac{e^{-\frac{(x_i - \sqrt{P_0} b)^2}{2\sigma^2}}}{d_i \sqrt{P_0}} - e^{-\frac{(x_i - \sqrt{P_0} a)^2}{2\sigma^2}} \frac{1}{d_i \sqrt{P_0}} \right) ds_i \right)^2,$$

then

$$E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial y_i^2} \right] = \sum_i \sum_l -\frac{\Delta_3}{p_{il}}.$$

4) J(2,3) element
According to (60), $\mathbf{J}(2,3)$ is

$$E \left[ \frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial x_i \partial y_j} \right] = \sum_i \sum_j -\frac{1}{p_{il}} \left[ \frac{\partial \ln p_{il}}{\partial x_i} \right] \left[ \frac{\partial \ln p_{il}}{\partial y_j} \right].$$  \hspace{1cm} (73)

Recall that $p_{il}$ is

$$p_{il}(\vec{x}, \theta) = \int_{\eta_l} \sqrt{\frac{P_0}{2\pi \sigma(b-a)}} \frac{1}{d_i} \frac{1}{\sqrt{P_0}} d \tau d s_i \left( 0 \leq l \leq L-1 \right).$$ \hspace{1cm} (74)

The partial derivative of $p_{il}$ with respect to $x_i$ is denoted as $A_i$, and the partial derivative of $p_{il}$ with respect to $y_i$ is denoted as $B_i$:

$$A_i = \frac{\partial \ln p_{il}(\vec{x}, \theta)}{\partial x_i} = \int_{\eta_l} \sqrt{\frac{P_0}{2\pi \sigma(b-a)}} \begin{cases} \frac{1}{d_i} \frac{1}{\sqrt{P_0}} & 
\end{cases} ds_i \hspace{1cm} (75)$$

$$B_i = \frac{\partial \ln p_{il}(\vec{x}, \theta)}{\partial y_i} = \int_{\eta_l} \sqrt{\frac{P_0}{2\pi \sigma(b-a)}} \begin{cases} \frac{1}{d_i} \frac{1}{\sqrt{P_0}} & 
\end{cases} ds_i \hspace{1cm} (76)$$

Now (73) can be expressed as

$$E \left[ \frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial x_i \partial y_j} \right] = \sum_i \sum_j \frac{A_i B_j}{p_{il}}.$$ \hspace{1cm} (77)

5) $\mathbf{J}(1,2)$ element

According to (60), $\mathbf{J}(1,2)$ is

$$E \left[ \frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial \mathbf{P}_0 \partial x_i} \right] = \sum_i \sum_j -\frac{1}{p_{il}} \left[ \frac{\partial \ln p_{il}}{\partial \mathbf{P}_0} \right] \left[ \frac{\partial \ln p_{il}}{\partial x_i} \right].$$ \hspace{1cm} (78)
Recall that \( p_{il} \) is
\[
p_{il}(\eta, \theta) = \int_{\eta}^{\eta_{l+1}} \frac{\sqrt{P_0}}{2\pi \sigma(b-a)} \int_{\eta_{d+a}}^{\eta_{d+b}} \frac{1}{\sigma^2} e^{-\frac{(x_i-x)^2}{2\sigma^2}} d\tau d\eta_i (0 \leq l \leq L-1). \tag{79}
\]
The partial derivative of \( p_{il} \) with respect to \( P_0 \) is denoted as \( A_2 \), and the partial derivative of \( p_{il} \) with respect to \( x_i \) is denoted as \( B_2 \) using
\[
A_2 = \frac{\partial \ln p_{il}(\eta, \theta)}{\partial P_0} = \int_{\eta}^{\eta_{l+1}} \frac{1}{2\sqrt{2\pi \sigma(b-a)}\sqrt{P_0}} \int_{\eta_{d+a}}^{\eta_{d+b}} \frac{1}{\sigma^2} e^{-\frac{(x_i-x)^2}{2\sigma^2}} d\tau ds_i + \frac{(d_i+a)e^{-\frac{(x_i-x)^2}{2\sigma^2}} - (d_i+b)e^{-\frac{(x_i-x)^2}{2\sigma^2}}}{2\sqrt{2\pi \sigma(b-a)}P_0} ds_i. \tag{80}
\]
\[
B_2 = \frac{\partial \ln p_{il}(\eta, \theta)}{\partial x_i} = \int_{\eta}^{\eta_{l+1}} \frac{\sqrt{P_0}}{2\pi \sigma(b-a)} \left[ \begin{array}{c} \frac{1}{\sigma^2} e^{-\frac{(x_i-x)^2}{2\sigma^2}} \left( \frac{x_i-x}{d_i} \right) P_0 \left( \eta_{d+a} \right) \\ \frac{1}{\sigma^2} e^{-\frac{(x_i-x)^2}{2\sigma^2}} \left( \frac{x_i-x}{d_i} \right) P_0 \left( \eta_{d+b} \right) \end{array} \right] ds_i. \tag{81}
\]
Now (78) can be expressed as
\[
E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial P_0 \partial x_i} \right] = \sum_i \sum_l -A_2 B_2 \frac{1}{p_{il}}. \tag{82}
\]

6) \( J(1,3) \) element

According to (60), \( J(1,3) \) is
\[
E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial P_0 \partial y_i} \right] = \sum_i \sum_l - \frac{1}{p_{il}} \left[ \frac{\partial \ln p_{il}}{\partial \theta} \right] \left[ \frac{\partial \ln p_{il}}{\partial \theta} \right]. \tag{83}
\]
Recall that \( p_{il} \) is
\[
p_{il}(\eta, \theta) = \int_{\eta}^{\eta_{l+1}} \frac{\sqrt{P_0}}{2\pi \sigma(b-a)} \int_{\eta_{d+a}}^{\eta_{d+b}} \frac{1}{\sigma^2} e^{-\frac{(x_i-x)^2}{2\sigma^2}} d\tau d\eta_i (0 \leq l \leq L-1). \tag{84}
\]
The partial derivative of \( p_{il} \) with respect to \( P_0 \) is denoted as \( A_i \), and the partial derivative of \( p_{il} \) with respect to \( y_i \) is denoted as \( B_i \) using

\[
A_i = \frac{\partial \ln p_{il}(\eta, \theta)}{\partial P_0} = \int_{\eta_{il}} \left[ \frac{1}{2\sqrt{2\pi}\sigma(b-a)\sqrt{P_0}} \int_{\sqrt{\frac{P_0}{d_i+b}}}^{\sqrt{\frac{P_0}{d_i+a}}} \frac{1}{\tau^2} e^{-\frac{(s_i-\tau)^2}{2\sigma^2}} d\tau \right] ds_i \tag{85}
\]

\[
B_i = \frac{\partial \ln p_{il}(\eta, \theta)}{\partial y_i} = \int_{\eta_{il}} \left[ \frac{\sqrt{P_0}}{\sqrt{2\pi}\sigma(b-a)} \left\{ \frac{e^{-\frac{(s_i-\sqrt{\frac{P_0}{d_i+a}})^2}{2\sigma^2}}}{d_i} \frac{1}{\sqrt{P_0}} (y_i - y_i) - \frac{e^{-\frac{(s_i-\sqrt{\frac{P_0}{d_i+b}})^2}{2\sigma^2}}}{d_i} \frac{1}{\sqrt{P_0}} (y_i - y_i) \right\} \right] ds_i \tag{86}
\]

Now (83) can be expressed as

\[
E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial P_0 \partial y_i} \right] = \sum_i \sum_l -\frac{A_i B_i}{p_{il}} \tag{87}
\]

Note that \( J(1,2) \) is equal to \( J(2,1) \), \( J(2,3) \) is equal to \( J(3,2) \), and \( J(1,3) \) is equal to \( J(3,1) \). Expressions for all elements of \( J \) have been determined.

**USING LOCALIZATION BASED ON THE LEM AS AN APPROXIMATE APPROACH FOR THE TVCEM AND CEM**

The LEM MLE approach has very limited application because most types of errors can not be described by the LEM. This section outlines a method for using the LEM MLE approach as an approximate approach when sensor position errors are described by the TVCEM or the CEM, which are more generally applicable models of sensor position errors. In the TVCEM and the CEM, sensors are located within a circle of
radius $R$ centered at the assumed sensor position (Figure 12). The distance between the actual position of sensor $i$ and the target can be expressed as the sum of two parts. The first part, $d_i$, is the distance between the fusion center and the $i$th sensor, and the second part, $h'_i$, is a random sensor position error. In the TVCEM, a sensor drifts from time to time within the circle, and therefore, values of $h'_i$ vary over time. In the CEM, the values of $h'_i$ do not vary over time.


Figure 12. The TVCEM and CEM representation.

Next, a scheme is outlined for using the LEM MLE approach as an approximate approach when sensor position errors are described by the TVCEM or the CEM. The distribution of $h'_i$ in the TVCEM or CEM is approximated by a variable $h_i$ that has zero mean and follows uniform distribution $h_i \in u[-b, b]$. The value of $b$ is determined by
\[ \text{var}(h'_i) = \text{var}(h_i) \]  \hspace{1cm} (88)

and

\[ \text{var}(h'_i) = b^2 / 3. \]  \hspace{1cm} (89)

The variance equation (88) is valid if \( d'_i >> R \). Assuming (88) is valid, (89) can be solved for the value of \( b \). If the distance from the target to any sensor is smaller than the value of \( b \), the distance is set to \( b \) in order to avoid numerical problems with the calculation of (50).

**Simulation Setup**

The appropriate value of \( \eta \) was determined using Monte Carlo simulations following the methods presented in Chapter II. The average NEES values used to determine the appropriate value of \( \eta \) were generated using 1000 Monte Carlo simulations in which targets were placed according to a random uniform distribution within a circle having radius equal to 25 and center at (12, 13). The 90% confidence interval for the average NEES value corresponding to 1000 Monte Carlo simulations and estimation of three parameters is \([2.8737 \, 3.1285]\). The average NEES value corresponding to \( \eta = 4 \) was within the confidence region, and the average NEES values for other values of \( \eta \) were outside the confidence region (Table 5). Therefore, \( \eta = 4 \) was used for the simulations.
The LEM MLE approach was tested for sensor position errors described by the LEM. Binary quantization, $\eta = 4$, $(x_t, y_t) = (12, 13)$, $P_0 = 10000$, and $\sigma^2 = 1$ were used.

In the LEM, the sensor drift was along a line from the target to the assumed sensor position. The drift $h_t$ followed a uniform distribution $h_t \in u[a, b]$. In the simulations, $a = 0$, and $b$ was varied from 0 to 30. Please note that the RMS estimation errors were computed for the estimates (58) and the CRLB was computed using (59). The RMS estimation errors were calculated based on 1000 Monte Carlo runs.

**TABLE 5**

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<th>Threshold, $\eta$</th>
<th>Average NEES</th>
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</tbody>
</table>

Moreover, the LEM MLE approach was tested as an approximate localization approach for sensor position errors described by the TVCEM and CEM. Binary quantization and the same $(x_t, y_t)$, $\eta$ and $P_0$ were used. In the simulations, $h'_t \in u[-b, b]$ was used, $R$ was varied in steps from 0 to 30, and $b$ was adjusted using (89) such that $\text{var}(h_t) = \text{var}(h'_t)$.
Results and Discussion

If sensor position errors followed the LEM, the RMS estimation errors (LEM MLE in Figure 13) given by the LEM MLE approach were very close to the CRLB. Moreover, the RMS estimation errors (Original MLE in Figure 13) produced by the MLE approach in [29], which does not account for sensor position errors, were higher than the RMS estimation errors given by the LEM MLE approach.

When the LEM MLE approach was used as an approximate approach for sensor position errors described by the TVCEM, RMS estimation errors were lower (LEM MLE in Figure 14) than RMS estimation errors given by the MLE approach in [29] (original MLE in Figure 14). Similarly, for sensor position errors described by the CEM, RMS estimation errors given by the LEM MLE approach (LEM MLE in Figure 15) were lower than RMS estimation errors given by the MLE approach in [29] (Original MLE in Figure 15), which did not account for sensor position errors.

Further research can be done to test the LEM MLE approach for sensor position errors described by other models including sensor position errors that follow Gaussian distributions. Moreover, if the sensor position errors can be characterized by PDFs, their PDFs can be included in the MLE framework to derive a more accurate estimator. Furthermore, the LEM MLE method could be tested in field experiments.
Summary

Although sensor position errors degrade localization performance of the MLE approach for energy-based target localization in [29], to the best of our knowledge, no method is available to alleviate performance degradation caused by sensor position errors. In this chapter, a LEM model was developed and was incorporated into the MLE approach for energy-based target localization. Simulation results showed that for sensor position errors described by the LEM, the TVCEM, or the CEM, RMS estimation errors for the LEM MLE approach were lower than RMS estimation errors given by the MLE approach in [29].


Figure 13. RMS estimation errors and CRLB for the LEM \((x_t, y_t) = (12,13)\) and \(\eta = 4\).
Figure 14. RMS estimation errors for the TVCEM \((x, y) = (12, 13)\) and \(\eta = 4\).

Figure 15. RMS estimation errors for the CEM \((x, y) = (12, 13)\) and \(\eta = 4\).
IV. ENERGY-ADJUSTABLE, FAULT-TOLERANT, AND CHANNEL-AWARE TARGET LOCALIZATION

The energy-based target localization method presented in [29] does not consider several factors that can degrade localization performance. One factor is imperfect communication channels, as discussed in Chapter II. Another factor is sensor faults. A third factor is sensor sleep. Sensors may randomly sleep to save energy. However, no approach is available to alleviate performance degradation caused by simultaneous effects of imperfect communication channels, sensor faults, and sensor sleep.

A combined model was developed to describe imperfect communication channels, sensor faults, and sensor sleep. The combined model was incorporated into the MLE framework in (8)-(11). An energy-adjustable, fault-tolerant, and channel-aware MLE approach for energy-based target localization can alleviate performance degradation caused by simultaneous imperfect communication channels, sensor faults, and sensor sleep. The sleep probability can be used to adjust the energy consumption of the WSN. The CRLB corresponding to the energy-adjustable, fault-tolerant, and channel-aware target localization method was derived. The CRLB serves as a performance criterion and as a predictor of localization performance.
Problem Setup

Sensor $i$ quantizes received signal $s_i$ using threshold $\eta$ to generate a measurement $m_i$. The quantization process is denoted as

$$m_i = \begin{cases} 0 & -\infty < s_i < \eta \\ 1 & \eta \leq s_i < \infty \end{cases} \quad (90)$$

where $m_i$ is either 0 or 1. As shown in [29], the conditional probability that $m_i$ is equal to $m$ is

$$p(m_i = m | \theta) = Q\left(\frac{\eta_i - a_i}{\sigma}\right) - Q\left(\frac{\eta_{i+1} - a_i}{\sigma}\right) \quad (0 \leq m \leq 1) \quad (91)$$

If sensors do not sleep, sensors do not fail, and the communication channels between sensors and the fusion center are perfect, the decisions received at the fusion center will be $M = [m_1, m_2, \ldots, m_n]$. However, sensors may be allowed to sleep to save energy. If sensors randomly sleep, the decisions sent by sensors will be $\tilde{M}$ (Figure 16). Moreover, sensors may fail. The sensor fault model is similar to the BSC model and is defined by failure probability $p_f$. If sensor faults occur, the decisions transmitted by sensors to the fusion center will be $\tilde{M}$ (Figure 16). Furthermore, communication channels between sensors and the fusion center may not be ideal. This is especially true when sensors are constrained by resources and cannot use sophisticated coding methods and transmission schemes, as explained in [30]. If communication channels are not perfect, the decisions received by the fusion center will be $\tilde{M}$ (Figure 16). In the following sections, these factors that degrade localization performance will be discussed in detail.
Sensor Sleep Model

In WSNs, saving energy is very important. Energy can be saved by allowing sensors to randomly sleep. In our sensor sleep scheme, if the decisions made by sensors are 0s, sensors will not send 0s to the fusion center. If the decisions are 1s, sensors will send 1s to the fusion center. Sensors may randomly sleep to further reduce energy consumption. Our sensor sleep scheme can be modeled, and the model is controlled by the only parameter, the sleep probability, \( p_s \). This sensor sleep model can save energy. However, allowing sensors to randomly sleep will degrade localization performance because sensors that should send 1s fail to send 1s, and the fusion center will assume the sensor decision is 0 if no decision is received. For the sensor sleep model shown in
Figure 16, the transition probabilities are

\[ p(\tilde{m}_i = 0 | m_i = 0) = 1 \quad (92) \]

\[ p(\tilde{m}_i = 1 | m_i = 0) = 0 \quad (93) \]

\[ p(\tilde{m}_i = 0 | m_i = 1) = p_s \quad (94) \]

\[ p(\tilde{m}_i = 1 | m_i = 1) = 1 - p_s . \quad (95) \]

Rayleigh Fading Channel Model with Soft-Decoder and Non-Coherent Receiver

The Rayleigh fading channel, which is important in WSNs, is a widely used model to describe a wireless communication channel. The Rayleigh fading channel with soft-decoder and non-coherent receiver is introduced in this section. In [30], a low-cost and widely used non-coherent detection technique, energy detection (ED), is employed. ED is used as the receiver for decisions transmitted over the Rayleigh fading channel. In [30], the received signal model from sensor \( i \) before ED is

\[ r_i = \begin{cases} v_i, & \bar{m}_i = 0 \\ h_i e^{j\phi}, & \bar{m}_i = 1 \end{cases} \quad (96) \]

where \( v_i \sim CN(0, 2\sigma_v^2) \), and \( h_i e^{j\phi} \sim CN(0,1) \) (unit power Rayleigh fading channel).

After ED, the observation model at the fusion center for the \( i \)th sensor is simply

\[ \bar{m}_i = |r_i|^2 \], where \(||\) indicates the magnitude of a complex number. Then, according to [30], the conditional PDF is
Finally, the conditional PDF in (97) was incorporated in the MLE framework.

\[
p(\tilde{m}_i | \bar{m}_i = 0) = \frac{1}{2\sigma_v^2} e^{-\frac{\tilde{m}_i}{2\sigma_v^2}}
\]

\[
p(\tilde{m}_i | \bar{m}_i = 1) = \frac{1}{1+2\sigma_v^2} e^{-\frac{\tilde{m}_i}{1+2\sigma_v^2}}.
\]

Combined Model

The combined model representing the sensor sleep model, faults, and communication channel imperfections can be determined by calculating the transition relation between \( m_i \) and \( \tilde{m}_i \) in Figure 16. For the BSC model of in Figure 16, the overall transition relation is

\[
p(\tilde{m}_i = 0 | m_i = 0) = 1 \times (1 - p_f)(1 - p_b) + 1 \times p_f \times p_b
\]

(98)

\[
p(\tilde{m}_i = 1 | m_i = 0) = 1 \times p_f \times (1 - p_b) + 1 \times (1 - p_f) \times p_b
\]

(99)

\[
p(\tilde{m}_i = 0 | m_i = 1) = p_s (1 - p_f)(1 - p_b) + (1 - p_s) p_f (1 - p_b) + (1 - p_s)(1 - p_f)p_b + p_s p_f p_b
\]

(100)

\[
p(\tilde{m}_i = 1 | m_i = 1) = (1 - p_s)(1 - p_f)(1 - p_b) + p_s p_f (1 - p_b) + p_s(1 - p_f)p_b + (1 - p_s)p_f p_b
\]

(101)

For a Rayleigh fading channel, the sleep model and fault model are combined.

The transition probabilities of the combined sleep and fault model are

\[
p(\bar{m}_i = 0 | m_i = 0) = 1 - p_f
\]

(102)

\[
p(\bar{m}_i = 1 | m_i = 0) = p_f
\]

(103)

\[
p(\bar{m}_i = 0 | m_i = 1) = (1 - p_s)p_f + p_s(1 - p_f)
\]

(104)
\[ p(\tilde{m}_i = 1|m_i = 1) = (1 - p_s)(1 - p_f) + p_fp_f. \]  

(105)

Then, the combined model in together with the relation in (97) is used to determine the relation between \( m_i \) and \( \tilde{m}_i \).

Since sensor sleep, sensor faults, and imperfect communication channels are assumed to be generated by independent processes, the probability that \( \tilde{m}_i \) is equal to \( m \) is

\[ p(\tilde{m}_i = m|\theta) = \sum_{m_i=0}^{L-1} p(\tilde{m}_i = m_{i}|m_i)p(m_i|\theta). \]  

(106)

For \( M = [\tilde{m}_1 \tilde{m}_2 \ldots \tilde{m}_M]^T \) received at the fusion center, the likelihood function is

\[ p(M|\theta) = \prod_{i=1}^{N} p(\tilde{m}_i|\theta) = \prod_{i=1}^{N} \left[ \sum_{m_{i}=0}^{L-1} p(\tilde{m}_i|m_i)p(m_i|\theta) \right]. \]  

(107)

The log-likelihood function is

\[ \ln p(M|\theta) = \sum_{i=1}^{N} \ln \left[ \sum_{m_{i}=0}^{L-1} p(\tilde{m}_i|m_i)p(m_i|\theta) \right]. \]  

(108)

The maximum likelihood estimator is

\[ \hat{\theta} = \max_{\theta} \ln p(M|\theta). \]  

(109)

For an unbiased \( \hat{\theta} \), the CRLB is

\[ E\{[\hat{\theta}(M) - \theta][\hat{\theta}(M) - \theta]^T\} \geq J^{-1} \]  

(110)

where

\[ J = -E \left[ \nabla_\theta \nabla_\theta^T \ln p(M|\theta) \right]. \]  

(111)
The Derivation of the CRLB for the Combined Model

The CRLB for the combined model of the BSC, sensor sleep, and sensor faults is derived. The CRLB for the combined model of the Rayleigh fading channel with non-coherent receiver, sensor sleep, and sensor faults is also derived.

The CRLB for the Combined Model of the BSC, Sensor Sleep, and Sensor Faults.

For a BSC model, the combined model is equivalent to a BSC, which is defined by $p(\tilde{m}_i | m_i)$ in (102-105). Therefore, the CRLB presented in [30] is used. For reference, the CRLB for the BSC model in [30] is presented below.

For the BSC, the FIM can be expressed as

$$[F(\theta)]_2 = E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial \theta_i \partial \theta_j} \right].$$

(112)

First, $J(1,1)$ is derived using

$$\frac{\partial^2 \ln p(M|\theta)}{\partial P_0^2} = \sum_{i=1}^{N} - \frac{1}{p^2(\tilde{m}_i | \theta)} \left[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0} \right]^2 + \frac{1}{p(\tilde{m}_i | \theta)} \left[ \frac{\partial^2 p(\tilde{m}_i | \theta)}{\partial P_0^2} \right]$$

(113)

and

$$J(1,1) = E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial P_0^2} \right] = \sum_{i=1}^{N} \sum_{m_i=1}^{L-1} \frac{1}{p(\tilde{m}_i | \theta)} \left[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0} \right]^2.$$  

(114)

In (114), $\frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0}$ is expressed as

$$\frac{\partial p(\tilde{m}_i | \theta)}{\partial P_0} = \sum_{m_i=0}^{1} p(\tilde{m}_i | m_i) \frac{\partial p(m_i | \theta)}{\partial P_0}.$$  

(115)
In (115), \( \frac{\partial p(m_i \mid \theta)}{\partial P_0} \) is

\[
\frac{\partial p(m_i = l \mid \theta)}{\partial P_0} = \frac{\partial}{\partial P_0} \left[ Q\left(\eta_{il} - a_i \right) - Q\left(\eta_{i(l+1)} - a_i \right) \right] = \frac{1}{2\sqrt{2\pi}\sigma \sqrt{P_0} d_i} e^{-\frac{(\eta_{il}-a_i)^2}{2\sigma^2}} - e^{-\frac{(\eta_{i(l+1)}-a_i)^2}{2\sigma^2}}. \tag{116}
\]

The derivation of (116) uses

\[
\frac{\partial Q(\eta_{il} - a_i)}{\partial P_0} = \frac{1}{2\sqrt{2\pi}\sigma \sqrt{P_0} d_i} e^{-\frac{(\eta_{il}-a_i)^2}{2\sigma^2}}. \tag{117}
\]

Now (115) can be expressed as

\[
\frac{\partial p(\tilde{m}_i \mid \theta)}{\partial P_0} = \sum_{m_i=0}^{1} p(\tilde{m}_i \mid m_i) \frac{1}{2\sqrt{2\pi}\sigma \sqrt{P_0} d_i} e^{-\frac{(\eta_{il}-a_i)^2}{2\sigma^2}} - e^{-\frac{(\eta_{i(l+1)}-a_i)^2}{2\sigma^2}}. \tag{118}
\]

In (118), the transition probability \( p(\tilde{m}_i \mid m_i) \) is defined in (102-105). Other elements of \( \textbf{J} \) can be derived similarly.

The CRLB for the Combined Model of the Rayleigh Fading Channel with Non-Coherent Receiver, Sensor Sleep, and Sensor Faults.

For the Rayleigh fading channel with a non-coherent receiver, the FIM is

\[
[F(\theta)]_{ij} = E \left[ \frac{\partial^2 \ln p(\textbf{M} \mid \theta)}{\partial \theta_i \partial \theta_j} \right]. \tag{119}
\]

1) First \( \textbf{J}(1,1) \) is derived. We have

\[
\frac{\partial^2 \ln p(\textbf{M} \mid \theta)}{\partial P_0^2} = \sum_{i=1}^{N} \frac{1}{p^2(\tilde{m}_i \mid \theta)} \left[ \frac{\partial p(\tilde{m}_i \mid \theta)}{\partial P_0} \right]^2 + \frac{1}{p(\tilde{m}_i \mid \theta)} \frac{\partial^2 p(\tilde{m}_i \mid \theta)}{\partial P_0^2}. \tag{120}
\]
and
\[
\mathbf{J}(1,1) = E \left[ \frac{\partial^2 \ln p \left( \mathbf{M} | \theta \right) }{\partial P_0^2} \right] = \sum_{i=1}^{N} \int_0^\infty -\frac{1}{p(\tilde{m}_i | \theta)} \left[ \frac{\partial p(\tilde{m}_i | \theta) }{\partial P_0} \right]^2 d\tilde{m}_i .
\] (121)

In (121), \( \frac{\partial p(\tilde{m}_i | \theta) }{\partial P_0} \) is
\[
\frac{\partial p(\tilde{m}_i | \theta) }{\partial P_0} = \sum_{m_i=0}^1 p(\tilde{m}_i | m_i) \frac{\partial p(m_i | \theta) }{\partial P_0} .
\] (122)

In (122), \( \frac{\partial p(m_i | \theta) }{\partial P_0} \) is
\[
\frac{\partial p(m_i = l | \theta) }{\partial P_0} = \frac{\partial}{\partial P_0} \left[ Q(\eta_i/a_i) - Q(\eta_{i(l+1)}/\sigma) \right] = \frac{1}{2\sqrt{2\pi}\sigma \sqrt{P_0 \sigma_i}} \left( e^{\frac{(\eta_i-a_i)^2}{2\sigma_i^2}} - e^{\frac{(\eta_{i(l+1)}-a_i)^2}{2\sigma_i^2}} \right) .
\] (123)

The derivation of (123) uses
\[
\frac{\partial Q(\eta_i/a_i)}{\partial p_0} = \frac{1}{2\sqrt{2\pi}\sigma \sqrt{P_0 \sigma_i}} e^{\frac{(\eta_i-a_i)^2}{2\sigma_i^2}} .
\] (124)

Now, (122) can be expressed as
\[
\frac{\partial p(\tilde{m}_i | \theta) }{\partial P_0} = \sum_{m_i=0}^1 p(\tilde{m}_i | m_i) \frac{1}{2\sqrt{2\pi}\sigma \sqrt{P_0 \sigma_i}} \left( e^{\frac{(\eta_i-a_i)^2}{2\sigma_i^2}} - e^{\frac{(\eta_{i(l+1)}-a_i)^2}{2\sigma_i^2}} \right) .
\] (125)

In (125), the transition probability \( p(\tilde{m}_i | m_i) \) is
\[
p(\tilde{m}_i | m_i = 0) = \frac{1}{2\sigma_i^2} f_{\chi_0^2}^{-\tilde{m}_i} + \frac{1}{1+2\sigma_i^2} f_{\chi_{\tilde{m}_i}^2}^{-\tilde{m}_i / 2} \times p_f
\] (126)

and
\[ p(\tilde{m}_i | m_i = 1) = \frac{1}{2\sigma_v^2} \times e^{-\frac{\tilde{m}_i}{2\sigma_v^2}} \times \left( p_x(1 - p_f) + (1 - p_x)p_f \right) \]
\[ + \frac{1}{1 + 2\sigma_v^2} \times e^{1 + 2\sigma_v^2} \left( (1 - p_x)(1 - p_f) + p_x p_f \right). \]

The final form of \( J(1, 1) \) is obtained by substituting (126) and (127) into (125).

2) \( J(2, 2) \) element

According to the definition,
\[ J(2, 2) = E \left[ \frac{\partial}{\partial x_i} \ln p(M|\theta) \right] = \sum_{m_i = 0}^X \int_0^\infty \left( \frac{\partial}{\partial x_i} \right) \left[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial x_i} \right] d\tilde{m}_i. \]  

In (128),
\[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial x_i} = \sum_{m_i = 0}^X p(\tilde{m}_i | m_i) \left( \frac{-\sqrt{P_0}(x_i - x_i)}{d_i^3 \sqrt{2\pi \sigma}} \right) \left( e^{-\frac{(\eta_i - a_i)^2}{2\sigma^2}} - e^{-\frac{(\eta_i - a_i)^2}{2\sigma^2}} \right). \]

For the derivation of (129),
\[ \frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_0}}{d_i} \right) = \frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_0}}{\sqrt{(x_i - x_i)^2 + (y_i - y_i)^2}} \right) = -\sqrt{P_0} \frac{1}{2} \frac{1}{d_i^3} 2(x_i - x_i) = -\sqrt{P_0} \frac{x_i - x_i}{d_i^3}, \]

\[ \frac{\partial Q(\eta_i - a_i)}{\sigma} = \frac{-1}{\sqrt{2\pi}} e^{-\frac{(\eta_i - a_i)^2}{2\sigma^2}}, \]

and
\[ \frac{\partial p(m_i = l | \theta)}{\partial x_i} = \frac{\partial}{\partial \eta_i} \left[ \frac{Q(\eta_i - a_i)}{\sigma} - Q(\eta_{i+1} - a_i) \right] \]
\[ = -\sqrt{P_0} \frac{x_i - x_i}{d_i^3 \sqrt{2\pi \sigma}} \left( e^{-\frac{(\eta_i - a_i)^2}{2\sigma^2}} - e^{-\frac{(\eta_{i+1} - a_i)^2}{2\sigma^2}} \right). \]

were used. In (132), the transition probability \( p(\tilde{m}_i | m_i) \) is
\[ p(\tilde{m}_i | m_i = 0) = \frac{1}{2\sigma_v^2} \times e^{-\frac{\tilde{m}_i}{2\sigma_v^2}} \times (1 - p_f) + \frac{1}{1 + 2\sigma_v^2} \times e^{1 + 2\sigma_v^2} \times p_f \]
\[ p(\tilde{m}_i | m_i = 1) = \frac{1}{2\sigma_v^2} x e^{-\frac{\tilde{m}_i^2}{2\sigma_v^2}} \times (p_x (1 - p_f) + (1 - p_x) p_f) + \frac{1}{1 + 2\sigma_v^2} x e^{1 + 2\sigma_v^2} ((1 - p_x)(1 - p_f) + p_x p_f) . \tag{134} \]

3) \( J(3,3) \) element

According to the definition,

\[ J(3,3) = E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial y_i^2} \right] = \sum_{i=1}^{N} \int_0^\infty \frac{1}{p(\tilde{m}_i | \theta)} \left[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial y_i} \right]^2 d\tilde{m}_i . \tag{135} \]

In (135), \( \frac{\partial p(\tilde{m}_i | \theta)}{\partial y_i} \) is

\[ \frac{\partial p(\tilde{m}_i | \theta)}{\partial y_i} = \sum_{m_i = 0}^1 p(\tilde{m}_i | m_i) - \sqrt{P_0} (y_i - y_i) \left( e^{\frac{(y_i - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_i - y_i)^2}{2\sigma^2}} \right) . \tag{136} \]

For the derivation of (136),

\[ \frac{\partial}{\partial y_i} \left( \frac{\sqrt{P_0}}{d_i} \right) = \frac{1}{\sqrt{(x_i - x_i)^2 + (y_i - y_i)^2}} \frac{\partial \sqrt{P_0}}{\partial y_i} \]
\[ = -\sqrt{P_0} \frac{1}{(d_i)^2} \frac{1}{2} \frac{d_i}{d_i} (y_i - y_i) = -\sqrt{P_0} \frac{(y_i - y_i)^2}{d_i^3} , \tag{137} \]

\[ \frac{\partial Q(\eta_i - a_i)}{\partial y_i} = -\frac{1}{\sqrt{2\pi}} e^{\frac{(\eta_i - a_i)^2}{2\sigma^2}} - \frac{1}{\sigma} \frac{\partial P_0}{\partial y_i} = -\frac{\sqrt{P_0}}{d_i^3} \frac{(y_i - y_i)^2}{\sigma^2} e^{\frac{(\eta_i - a_i)^2}{2\sigma^2}} , \tag{138} \]

and

\[ \frac{\partial p(m_i = 1 | \theta)}{\partial y_i} = \frac{\partial}{\partial y_i} \left[ Q(\eta_i - a_i) - Q(\eta_{(i+1)} - a_i) \right] \]
\[ = -\frac{\sqrt{P_0}}{d_i^3} \frac{(y_i - y_i)}{\sqrt{2\pi\sigma}} \left( e^{\frac{(y_i - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_{(i+1)} - a_i)^2}{2\sigma^2}} \right) . \tag{139} \]
were used. In (136), the transition probability \( p(\hat{m}_t|m_t) \) is

\[
p(\hat{m}_t|m_t = 0) = \frac{1}{2\sigma_v^2} e^{-\hat{m}_t^2/2\sigma_v^2} \times (1 - p_f) + \frac{1}{1 + 2\sigma_v^2} e^{+1/2\sigma_v^2} \times p_f
\]

(140)

and

\[
p(\hat{m}_t|m_t = 1) = \frac{1}{2\sigma_v^2} e^{-\hat{m}_t^2/2\sigma_v^2} \times (p_f(1 - p_f) + (1 - p_f)p_f)
\]

\[+ \frac{1}{1 + 2\sigma_v^2} e^{1/2\sigma_v^2} ((1 - p_f)(1 - p_f) + p_f p_f).
\]

(141)

4) \( J(2,3) \) element

According to the definition,

\[
J(2,3) = E \left[ \frac{\partial^2 \ln p(M|\theta)}{\partial x_i \partial y_i} \right] = \sum_{t=1}^{N} \int_{0}^{\infty} \frac{1}{p(\hat{m}_t|\theta)} \left[ \frac{\partial p(\hat{m}_t|\theta)}{\partial x_i} \frac{\partial p(\hat{m}_t|\theta)}{\partial y_i} \right] d\hat{m}_t \quad (142)
\]

In (142), \( \frac{\partial p(\hat{m}_t|\theta)}{\partial x_i} \) and \( \frac{\partial p(\hat{m}_t|\theta)}{\partial y_i} \) are

\[
\frac{\partial p(\hat{m}_t|\theta)}{\partial x_i} = \sum_{m_i=0}^{1} p(\hat{m}_t|m_i) - \sqrt{P_0} (x_i - x_i) \frac{(\eta_i - a_i)^2}{2\sigma^2} - e^{-\frac{(\eta_i - a_i)^2}{2\sigma^2}}
\]

(143)

and

\[
\frac{\partial p(\hat{m}_t|\theta)}{\partial y_i} = \sum_{m_i=0}^{1} p(\hat{m}_t|m_i) - \sqrt{P_0} (y_i - y_i) \frac{(\eta_i - a_i)^2}{2\sigma^2} - e^{-\frac{(\eta_i - a_i)^2}{2\sigma^2}}
\]

(144)

The transition probabilities in (143) and (144) are the same as those in (133) and (134), respectively. For the derivation of (143),

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\[
\frac{\partial}{\partial x_i} \left( \sqrt{\frac{P_o}{d_i}} \right) = \frac{\partial}{\partial x_i} \left( \frac{\sqrt{P_o}}{\sqrt{(x_i - x_i)^2 + (y_i - y_i)^2}} \right) \\
= -\sqrt{P_o} \frac{1}{(d_i)^2} \cdot 2(x_i - x_i) \\
= -\frac{\sqrt{P_o}}{d_i^3} (x_i - x_i)
\]
(145)

\[
\frac{\partial Q\left( \frac{\eta_{il} - a_i}{\sigma} \right)}{\partial x_i} = \frac{-1}{\sqrt{2\pi}} e^{-\frac{(\eta_{il} - a_i)^2}{2\sigma^2}} - \frac{1}{\sigma} \frac{\sqrt{P_o}}{d_i^3} (x_i - x_i) \\
= -\frac{\sqrt{P_o}}{d_i^3 \sqrt{2\pi} \sigma} e^{\frac{(\eta_{il} - a_i)^2}{2\sigma^2}}
\]
(146)

and

\[
\frac{\partial p(m_i = l|\theta)}{\partial x_i} = \frac{\partial}{\partial p_0} \left[ Q\left( \frac{\eta_{il} - a_i}{\sigma} \right) - Q\left( \frac{\eta_{il(i+1)} - a_i}{\sigma} \right) \right] \\
= \frac{-\sqrt{P_o}}{d_i^3 \sqrt{2\pi} \sigma} \left( e^{\frac{(\eta_{il} - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_{il(i+1)} - a_i)^2}{2\sigma^2}} \right)
\]
(147)

were used. For the derivation of (144),

\[
\frac{\partial}{\partial y_i} \left( \sqrt{\frac{P_o}{d_i}} \right) = \frac{\partial}{\partial y_i} \left( \frac{\sqrt{P_o}}{\sqrt{(x_i - x_i)^2 + (y_i - y_i)^2}} \right) \\
= -\sqrt{P_o} \frac{1}{(d_i)^2} \cdot 2(y_i - y_i) - \frac{\sqrt{P_o}}{d_i^3} (y_i - y_i)
\]
(148)

\[
\frac{\partial Q\left( \frac{\eta_{il} - a_i}{\sigma} \right)}{\partial y_i} = \frac{-1}{\sqrt{2\pi}} e^{-\frac{(\eta_{il} - a_i)^2}{2\sigma^2}} - \frac{1}{\sigma} \frac{\sqrt{P_o}}{d_i^3} (y_i - y_i) \\
= -\frac{\sqrt{P_o}}{d_i^3 \sqrt{2\pi} \sigma} e^{\frac{(\eta_{il} - a_i)^2}{2\sigma^2}}
\]
(149)

and
\[ \frac{\partial p(m_i = l|\theta)}{\partial y_i} = \frac{\partial}{\partial \hat{p}_0} \left[ Q(\eta_i - a_i) \right] - \frac{\partial}{\partial \hat{p}_0} \left[ Q(\eta_{i(l+1)} - a_i) \right] \]
\[ = -\frac{P_i(y_i - y_i)}{d_i^3 \sqrt{2\pi \sigma}} (e^{\frac{(\eta_i - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_{i(l+1)} - a_i)^2}{2\sigma^2}}) \] (150)

were used.

5) **J**(1, 2) element

According to the definition,
\[ \mathbf{J}(1,2) = E \left[ \frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial \hat{p}_0 \partial x_i} \right] = \sum_{i=1}^{\infty} \frac{1}{p(\hat{m}_i|\theta)} \left[ \frac{\partial p(\hat{m}_i|\theta)}{\partial \hat{p}_0} \right] \left[ \frac{\partial p(\hat{m}_i|\theta)}{\partial x_i} \right] d\hat{m}_i \] (151)

In (151), \( \frac{\partial p(\hat{m}_i|\theta)}{\partial \hat{p}_0} \) and \( \frac{\partial p(\hat{m}_i|\theta)}{\partial x_i} \) are defined as
\[ \frac{\partial p(\hat{m}_i|\theta)}{\partial \hat{p}_0} = \sum_{m_i=0}^{1} p(\hat{m}_i|m_i) \frac{1}{2\sqrt{2\pi \sigma} \sqrt{\hat{p}_0 d_i}} (e^{\frac{(\eta_i - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_{i(l+1)} - a_i)^2}{2\sigma^2}}) \] (152)

and
\[ \frac{\partial p(\hat{m}_i|\theta)}{\partial x_i} = \sum_{m_i=0}^{1} p(\hat{m}_i|m_i) \frac{\sqrt{\hat{p}_0 (x_i - x_i)}}{d_i^3 \sqrt{2\pi \sigma}} (e^{\frac{(\eta_i - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_{i(l+1)} - a_i)^2}{2\sigma^2}}) \] (153)

The transition probabilities in (152) and (153) are the same as those in (133) and (134), respectively.

6) **J**(1, 3) element

According to the definition,
\[ \mathbf{J}(1,3) = E \left[ \frac{\partial^2 \ln p(\mathbf{M}|\theta)}{\partial \hat{p}_0 \partial y_i} \right] = \sum_{i=1}^{\infty} \frac{1}{p(\hat{m}_i|\theta)} \left[ \frac{\partial p(\hat{m}_i|\theta)}{\partial \hat{p}_0} \right] \left[ \frac{\partial p(\hat{m}_i|\theta)}{\partial y_i} \right] d\hat{m}_i \] (154)

In (154), \( \frac{\partial p(\hat{m}_i|\theta)}{\partial \hat{p}_0} \) and \( \frac{\partial p(\hat{m}_i|\theta)}{\partial y_i} \) are
\[ \frac{\partial p(\hat{m}_i|\theta)}{\partial \hat{p}_0} = \sum_{m_i=0}^{1} p(\hat{m}_i|m_i) \frac{1}{2\sqrt{2\pi \sigma} \sqrt{\hat{p}_0 d_i}} (e^{\frac{(\eta_i - a_i)^2}{2\sigma^2}} - e^{\frac{(\eta_{i(l+1)} - a_i)^2}{2\sigma^2}}) \] (155)

and
The transition probabilities in (155) and (156) are the same as those in (133) and (134), respectively. Note that \( J(3, 2) = J(3, 2) \), \( J(1, 2) = J(2, 1) \) and \( J(1, 3) = J(3, 1) \).

Expressions for all elements of \( J \) have been determined.

**Simulation Setup**

The appropriate value of \( \eta \) was determined using Monte Carlo simulations following the methods presented in Chapter II. The average NEES values used to determine the appropriate value of \( \eta \) were generated by using 1000 Monte Carlo simulations in which targets were placed according to a random uniform distribution within a circle having radius equal to 25 and centered at \((12, 13)\). The 95% confidence interval for the average NEES value corresponding to 1000 Monte Carlo simulations and estimation of three parameters is \([2.8500, 3.1537]\). The average NEES values corresponding to \( \eta = 2 \) and \( \eta = 3 \) for \( p_s = p_f = p_b = 0.001 \) and \( p_s = p_f = p_b = 0.01 \) were within the confidence region and the average NEES values for other values of \( \eta \) were outside the confidence region (Table 6). However, \( \eta = 2 \) saturates most sensors in the field and too many saturated sensors in the field can result in biased estimates. Therefore, \( \eta = 3 \) was used for the simulations.
TABLE 6

AVERAGE NEES VALUES GIVEN BY THE ENERGY-ADJUSTABLE, FAULT-TOLERANT, AND
CHANNEL-AWARE TARGET LOCALIZATION METHOD (DIFFERENT $p_s = p_f = p_b$
COMBINATIONS WITH DIFFERENT THRESHOLDS, AND 1000 TARGETS $(x_i, y_i)$
DISTRIBUTED WITHIN A CIRCLE CENTERED AT (12, 13) WITH RADIUS 25)

<table>
<thead>
<tr>
<th>Probability $p_s = p_f = p_b$</th>
<th>Threshold, $\eta$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
<th>$\eta = 3$</th>
<th>$\eta = 4$</th>
<th>$\eta = 5$</th>
<th>$\eta = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0001$</td>
<td></td>
<td>3.3203</td>
<td>3.0774</td>
<td>3.1287</td>
<td>3.2169</td>
<td>3.2374</td>
<td>3.1734</td>
</tr>
<tr>
<td>$0.001$</td>
<td></td>
<td>3.3016</td>
<td>3.1233</td>
<td>3.1110</td>
<td>3.2234</td>
<td>3.2736</td>
<td>3.7373</td>
</tr>
<tr>
<td>$0.01$</td>
<td></td>
<td>7.6696</td>
<td>3.4420</td>
<td>3.7186</td>
<td>5.6036</td>
<td>11.2250</td>
<td>20.6615</td>
</tr>
</tbody>
</table>

The $p_s$, $p_f$, and $p_b$ values affect localization performance. In simulations,
values of $p_s = p_f = p_b$ were varied to demonstrate their overall effect on RMS errors in
the estimates computed using (109) and to demonstrate their effect on the CRLB (110).
Then, $p_f = p_b$ was fixed, and $p_s$ was varied to demonstrate the effect of $p_s$.
Moreover, for the Rayleigh fading channel with a non-coherent receiver, the SNR value
of the channel was varied to demonstrate its effect on localization performance. The RMS
estimation errors were calculated based on 1000 Monte Carlo runs.

Results and Discussion

When the $p_s$, $p_f$, and $p_b$ values were low, the RMS estimation errors were
close to the CRLB (Figure 17). When the $p_s$, $p_f$, and $p_b$ values were large, the RMS
estimation errors were much greater than the CRLB (Figure 17). For BSC models, the
$p_s$ values played an important role (Figure 18); low $p_s$ values resulted in lower RMS
estimation errors, and large $p_s$ values resulted in greater RMS estimation errors.
Moreover, if the channels between the sensors and the fusion center were Rayleigh fading channels with non-coherent receivers, the SNR values played an important role (Figure 19). Low SNR values resulted in low RMS estimation errors and large SNR values resulted in large RMS estimation errors.

Future work might consider testing the energy-adjustable, fault-tolerant, and channel-aware MLE approach for energy-based target localization for different target positions. Moreover, simulations might be repeated for different communication channel models and sensor sleep models. Other factors that can affect localization performance, such as sensor position uncertainty, might also be included into the MLE framework.

![Figure 17. RMS estimation errors and CRLB (BSC channel, \( \eta = 3 \)).](image-url)
Figure 18. RMS estimation errors and CRLB (BSC channel, $p_f = p_b = 0.01$, $p_s$ varies, $(x_s, y_s) = (12, 13)$ and $\eta = 3$).

Figure 19. RMS estimation errors and CRLB (Rayleigh channel and non-coherent receiver, $p_s = p_f = 0.001$, $(x_s, y_s) = (12, 13)$ and $\eta = 3$).
Conclusion

Simulations demonstrated that the performance of the energy-based target localization method presented in [29] degraded dramatically when sensors failed, the sensor sleep scheme was used, and communication channels were not perfect. Our energy-adjustable, fault-tolerant, and channel-aware MLE approach for energy-based target localization provided robust performance.
V. BALANCING ENERGY CONSUMPTION AND PERFORMANCE OF ENERGY-BASED TARGET LOCALIZATION USING MULTI-OBJECTIVE OPTIMIZATION

The operating life of WSNs depends on energy consumption. Therefore, saving energy is important in WSNs. The sensor sleep scheme can save energy by allowing sensors to randomly sleep. However, sleeping sensors cannot send decisions to the fusion center, and missing decisions from sensors will degrade localization performance. Therefore, energy is saved at the expense of localization performance, and there is a tradeoff between saving energy and localization performance. The problem of balancing the energy consumption and localization performance using the sleep probability $p_s$ is a multi-objective optimization problem. Our approach to address this problem is to model energy consumption of a WSN and localization performance and use a nondominated sorting genetic algorithm, NSGA-II, to balance energy consumption and localization performance. The NSGA-II is a fast and elitist multi-objective genetic algorithm, and in most applications, it can find a Pareto-front. Details about NSGA-II can be found in [38]. The designer can use the Pareto-front found by NSGA-II to choose desired localization performance and acceptable energy consumption.

The energy consumption function and the localization performance function for a one-dimensional sensor array are presented first. Then, the energy consumption function
and the performance function for two-dimensional sensor arrays are presented. The multi-objective optimization algorithm is briefly reviewed. Finally, the simulation setup is discussed. Results are provided and discussed. The main content of this section has been published in [39].

Models of Energy Consumption and Localization Performance for One-Dimensional Sensor Arrays

In the following subsections, the performance function and energy consumption model are presented for one-dimensional and two-dimensional sensor arrays, respectively.

*Energy Consumption Model for One-Dimensional Sensor Arrays*

The energy consumption model includes the energy used by non-sleeping sensors and the energy used by sleeping sensors. For the one-dimensional sensor array in Figure 20, the global energy consumption $E_T$ in a WSN with a specific target location and parallel decision transmission is

$$E_T = \sum_{\text{all } \mathbf{u}} E_c(\mathbf{u}) \left[ p_0 \prod_{i=1}^{N} p(u_i|H_0) + p_1 \prod_{i=1}^{N} p(u_i|x,y,H_1) \right]$$

(157)

where $H_1$ is the probability of target presence, $H_0$ is the probability of target absence, $\mathbf{u}$ is the vector of sensor decisions, $u_i$ is the decision from sensor $i$, $p_0$ is the *a priori* probability of $H_0$, and $p_1$ is the *a priori* probability of $H_1$, as described in [33]. The probability for sensor $i$ to have decision $u_i$, if the target is absent, is $p(u_i|H_0)$. 

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The probability for sensor $i$ to have decision $u_i$, if a target is located at $(x, y)$, is $p(u_i | x, y, H_1)$. In (157), $E_c(u)$ is the energy required for sensors to send the decision vector $u$ to the fusion center.

Figure 20. One-dimensional sensor array.

Next, the elements of $E_c(u)$ are defined. If sensor $i$ transmits $m$ bits of information to the fusion center, the energy consumption $E_{TX}$ is

$$E_{TX}(m, d_{f,i}) = E_{elec} \times m + \varepsilon_{amp} \times m \times d_{f,i}^2$$

(158)

where $E_{elec} = 50 \, \text{uJ/bit}$, $\varepsilon_{amp} = 100 \, \text{nJ/bit/m}^2$, and $d_{f,i}$ is the distance between the $i$th sensor and the fusion center. The model (158) is similar to the models in [33], [39], and [40]. Using (158), the energy consumption for sensors to transmit the decision vector $u$ to the fusion center is

$$E_c(u) = \sum_{i=1}^{N} E_{TX}(u_i, d_{f,i}) .$$

(159)

If the target is assumed to be always present, (157) can be simplified as
\[ E_r = \sum_{\text{all } u} E_r(u) \left[ \prod_{j=1}^{N} p(u_j | x, y, H_1) \right]. \]  \hspace{1cm} (160)

It is assumed in this research that the sensing cycle time is 1 millisecond, which means that sensors make 1,000 detections in one second. If \( A \) is the energy used by an awake sensor in 1 microsecond, and \( B \) is the energy used by a sensor during sleep in 1 microsecond, the overall energy consumption of all sensors in 1 microsecond is

\[ E_a = (1 - p_s)(NA + E_r) + Np_sB, \]  \hspace{1cm} (161)

where \( N \) is the total number of sensors in a WSN and \( p_s \) is the probability with which sensors randomly sleep. The energy consumption function is \( f_i = E_a(p_s) \) where the sleep probability \( p_s \) is the only input of \( f_i \). Note that the calculation of \( E_r \) using (160) involves all combinations of the decision vector \( u \), and the number of all combination increases exponentially with \( N \). Therefore, if \( N \) is a large number, the calculation of (160) will become infeasible.

**The Performance Function for One-Dimensional Sensor Arrays**

For a one-dimensional sensor array, in order to avoid numerical problems with computing \( \hat{P}_0 \), \( P_0 \) is assumed to be known, and only the X-location of the target is estimated. If the estimation is unbiased, the FIM for the X-location estimate is

\[ J_1 = [J_{11}]. \]  \hspace{1cm} (162)

For a one-dimensional sensor array, the sensor sleep scheme in Figure 16 is used and the
details of the method used to calculate $J_1$ are similar to those in [30]. The performance function

$$V_1(p_s) = J_1^{-1}$$

(163)

was selected to correspond to the CRLB, which is the lower bound for the RMS estimation error for the X-location. For a one-dimensional sensor array, $f_2 = V_1(p_s)$ is used as the performance function, and $p_s$ is the only input. The problem of balancing (161) and (163) is a multi-objective optimization problem, which is solved by the NSGA-II method.

Models of Energy Consumption and Localization Performance for Two-Dimensional Sensor Arrays

The energy consumption function and the performance function for two-dimensional sensor arrays are presented in this section. As mentioned earlier, the calculation of the energy using (160) is computationally intensive if the number of sensors is large. For two-dimensional sensor arrays that have a large number of sensors, an approximation method for computing energy will be used to reduce computation cost.

*Energy Consumption Model for Two-Dimensional Sensor Arrays*

The following steps were used in deriving the energy consumption model for two-dimensional sensor arrays. In the following steps, it is assumed that $w = 0$, which means the signals received by sensors are noise free.
1) Assume that all sensors use the same \( \eta \) and make binary decisions. Then, only awake sensors within the circular of radius \( R \) centered at the target location will fire. The \( \eta \) can be used to determine the range \( R \) within which all sensors should fire. The range \( R \) is determined by

\[
R = \frac{\sqrt{P_0}}{\eta}.\]  

(164)

The number of sensors within the range \( R \) is denoted as \( N_{\text{range}} \).

2) Calculate the energy consumption for each sensor in the fired field to transmit one bit information to the fusion center using (158).

3) Calculate the average energy consumption for the \( N_{\text{range}} \) sensors within the range \( R \) to send one bit of decision to the fusion center using

\[
E_{\text{average}} = \frac{\sum_{\text{in Sensors in the range}} E_{\text{TX}}(u_i, d_{f,i})}{N_{\text{range}}}.\]  

(165)

4) Calculate the overall energy consumption in 1 microsecond using

\[
E_b = N_{\text{range}}(1 - \rho_s)(A + E_{\text{average}}) + (N - N_{\text{range}})(1 - \rho_s)A + N\rho_sB.\]  

(166)

The first part of (166) is the energy used by the awake sensors located within the range \( R \); these sensors will fire and transmit their 1 decisions to the fusion center. The second part is the energy used by the awake sensors located outside the range \( R \); these sensors will not fire, and therefore, they do not transmit their 0 decisions to the fusion center. The third part is the energy used by the sleeping sensors in the whole
field; these sensors do not make measurements, and they do not transmit their decisions to the fusion center.

The Performance Function for Two-Dimensional Sensor Arrays

The parameters $\theta = [P_0 x_t y_t]^T$ are estimated for two-dimensional sensor arrays. Therefore, the CRLB matrix is a $3 \times 3$ matrix. Usually, only the $x_t$ estimation errors and the $y_t$ estimation errors are the most important, as explained in [29]. Therefore, the sum of the $(2, 2)$ and $(3, 3)$ elements of $J^{-1}$ is used as the performance function

$$V_2 = J^{-1}(2, 2) + J^{-1}(3, 3).$$

The method used to calculate $J$ can be found in [30]. For two-dimensional sensor arrays, the NSGA-II method was used to balance the energy consumption function in (166) and localization performance function in (167).

Multi-Objective Optimization

Multi-objective optimization is an important research area and has many applications. By definition, multi-objective optimization algorithms can jointly optimize several conflicting goals subject to some constraints, as described in [41]. For example, in the business area, one can use multi-objective optimization algorithms to maximize profit and minimize cost. In the construction area, multi-objective optimization algorithms can be used to save construction material while improving the quality of a building. In the target detection area, multi-objective optimization algorithms can be used to increase the
detection rate while decreasing the false alarm rate. In [33], a performance function and an energy consumption function were presented, and a multi-objective optimization algorithm was used to jointly optimize the performance function and the energy consumption function for distributed detection in WSNs.

Usually, for a multi-objective optimization problem, no single solution can simultaneously reach the maximum or minimum value for every objective. The solution to a multi-objective optimization problem is a Pareto-front, which means that one objective cannot be improved without jeopardizing another objective [41-43].

The simplest solution for a multi-objective optimization problem is to assign a weight to every objective and convert the multi-objective optimization problem to a single-objective optimization problem, as described in [43]. However, in this method, one has to choose weights for all objectives. The performance of the final solution highly depends on the weights, as explained in [41] and [43]. NSGA-II, a modern optimization method based on evolutionary algorithms, can avoid this problem. It can simultaneously acquire $M$ Pareto optimal solutions according to [38]. The NSGA-II algorithm is used to jointly optimize the performance and energy consumption in a WSN with either a one-dimensional sensor array or two-dimensional sensor arrays.

Simulation Setup

The solution to a multi-objective optimization problem is a Pareto-front, which can be used to determine the appropriate energy consumption and localization
performance of the energy-based target localization method. The Pareto-front was generated for a one-dimensional sensor array by using NSGA-II to balance the energy consumption function (161) and target localization performance function (163). Similarly, the Pareto-front was generated for two-dimensional sensor arrays by using NSGA-II to balance the energy consumption function (166) and localization performance function (167).

For a one-dimensional sensor array, eight sensors were uniformly distributed (Figure 20). The fusion center was placed at (100, 0), and the target was located at \((x_t, y_t) = (0, 0)\). For the simulations, \(P_0 = 100\), \(\sigma^2 = 1\), \(A = 70e^{-6}\) joule, and \(B = 20e^{-6}\) joule were used. All sensors employed the same \(\eta\), which was changed from 1.5 to 2 and finally to 2.5. Sleep probability \(p_s\) was varied from 0.01 to 0.15. The value of \(P_0\) was assumed to be known by the fusion center. Only \(x_t\) was estimated. The generation number of the NSGA-II algorithm was 20, and the population of the NSGA-II algorithm was 50. Because target position affects the energy function and the localization performance function, Pareto-fronts were generated for different target positions.

For the two-dimensional sensor array, 441 sensors were placed in the sensor field (Figure 1), and the fusion center was placed at (100, 100). In the simulations, \(\eta = 5\) for all sensors, \(P_0 = 10,000\), \(\sigma^2 = 1\), \(A = 70e^{-6}\) joule, \(B = 20e^{-6}\) joule, and \((x_t, y_t) = (4, 4)\) were used. The fusion center estimates \(\theta = [p_0, x_t, y_t]\) and the sleep
probability $p_s$ was varied from 0.01 to 0.15. The population number of the NSGA-II algorithm was 50, and the generation number of the NSGA-II algorithm was 20. The Pareto-front was also generated for a different target position $(x_t, y_t) = (10, 10)$.

Results and Discussion

Energy consumption is important in the design of WSNs. For the one-dimensional sensor array, the energy consumption increased as $\eta$ decreased (Figure 21). This is because more sensors are fired and more sensors need to transmit decisions to the fusion center if the $\eta$ value is low. As for the effect of target position on the Pareto-front, energy consumption did not change much as the target position changed (Figure 21 and Figure 22). In contrast, localization performance changed as the target position changed (Figure 21 and Figure 22).

For the two-dimensional sensor array, the energy consumption also increased as $\eta$ decreased (Figure 23). This is because the higher $\eta$ values made fewer sensors fire, and fewer sensors transmitted decisions to the fusion center. As for the effect of target position on the Pareto-front, energy consumption did not change much as the target position changed (Figure 23 and Figure 24). In contrast, localization performance changed as the target position changed (Figure 23 and Figure 24). The Pareto-front is very useful. For example, if the desired performance is $3 \ m^2$ and $\eta = 2$, the energy consumption will be about 0.74J according to Figure 21. If $\eta = 2.5$, the energy consumption will be about 0.66J. Therefore, the required performance could be achieved
with lower energy by using higher \( \eta \) values.

![Figure 21. Pareto optimal front derived from balancing \( E_u \) and \( V_1 \) for a one-dimensional sensor array (a: \( \eta = 1.5 \), b: \( \eta = 2 \), c: \( \eta = 2.5 \), and \( (x, y) = (0, 0) \)).](image)

Energy can be saved by allowing more sensors to sleep. However, sleeping sensors will not make detections and they will not send decisions to the fusion center; missing decisions from sensors will degrade localization performance. In contrast, if fewer sensors sleep, energy consumption is higher and localization performance is better. The range over which \( p_s \) can vary is limited. If \( p_s \) is too high, not enough sensors will send decisions to the fusion center, and theoretical basis for the performance functions (163) and (167) will not be valid.
Figure 22. Pareto optimal front derived from balancing $E_v$ and $V_1$ for a one-dimensional sensor array (a: $\eta = 1.5$, b: $\eta = 2$, c: $\eta = 2.5$, and $(x, y) = (1, 0)$).

Figure 23. Pareto optimal front derived from balancing $E_b$ and $V_2$ for a two-dimensional sensor array (a: $\eta = 4$, b: $\eta = 5$, c: $\eta = 6$, and $(x, y) = (4, 4)$).
Figure 24. Pareto optimal front derived from balancing $E_b$ and $V_z$ for a two-dimensional sensor array (a: $\eta = 4$, b: $\eta = 5$, c: $\eta = 6$, and $(x_t, y_t) = (10, 10)$).

The Pareto-fronts (Figures 21-24) showed that localization performance changed more than energy consumption as target position was varied. The system designer can predict the changes of energy consumption and localization performance according to the Pareto-fronts.

In future work, the effect of $A$ and $B$ values on energy consumption and localization performance can be studied. The method to calculate energy consumption for two-dimensional sensor arrays is an approximate method. Further research can focus on developing a more accurate method to calculate energy consumption for two-dimensional sensor arrays.
Summary

In this section, NSGA-II was used to balance localization performance and energy consumption in WSNs for either one-dimensional or two-dimensional sensor arrays by choosing the sleep probability $p_s$. Using the Pareto-front, one can choose the desired localization performance with acceptable energy consumption. Moreover, the system designer can use the Pareto-fronts corresponding to typical target positions to predict the energy consumption and localization performance for a new target position.
VI. CONCLUSION

In this dissertation, several factors that can affect the localization performance of the MLE approach for energy-based target localization using quantized data in [29] were discussed and modeled. These models were incorporated into the MLE framework either separately or simultaneously to improve localization performance. In addition, a sensor sleep model was developed in which the sensor sleep probability can be used to adjust energy consumption of the sensor network. Research in this dissertation considered three main objectives.

First, the performance of the energy-based target localization method in [29] was improved by addressing some factors that can degrade localization performance, such as sensor faults and imperfect communication channels, including multi-hop communication channels. A combined model that considers the effects of several factors that act simultaneously to degrade localization performance was developed. Models were derived to account for simultaneous communication channel imperfections, sensor faults and sensor sleep. The combined model was incorporated into the MLE framework. Simulation showed that the RMS estimation errors given by the energy-adjustable, fault-tolerant, and channel-aware target localization method were close to the CRLB. Another factor can degrade localization performance is sensor position uncertainty. In the
work in [29] and [30], the fusion center localizes a target using decisions from sensors whose positions are accurately known. However, if the fusion center does not have accurate knowledge of sensor positions, localization performance suffers. Models for three types of sensor position uncertainty were developed, and the LEM was incorporated into the MLE framework. The LEM MLE approach for target localization improved localization performance for sensor position uncertainty described by the LEM, the TVCEM or the CEM. Simulation results showed that RMS estimation errors given by the LEM MLE approach for target localization were lower than the RMS estimation errors given by the MLE approach in [29] for sensor position uncertainty described by the LEM, the TVCEM or the CEM.

Second, a sensor sleep scheme in which sensors are allowed to sleep at random was developed to save energy. In this scheme, the sleep probability can be used to adjust the energy consumption of the WSN. The MLE approach for target localization was extended to accommodate sensors that are allowed to sleep at random. This MLE approach incorporated a sensor sleep model. Simulation results showed that the RMS estimation errors given by this MLE approach were close to the CRLB.

Finally, a multi-objective optimization method was used to balance energy consumption and localization performance in WSNs using the sleep scheme. The multi-objective optimization method used was NSGA-II, which can find the Pareto-front for most multi-objective optimization problems. Pareto-fronts were generated for different $\eta$ values and target positions. The system designer can use the Pareto-fronts
corresponding to typical target positions to predict the energy consumption and localization performance for a new target position.
LIST OF REFERENCES


